

$$x \overbrace{L}^{\nu} = x^{\mu} \underset{\mu}{L}^{\nu} + L^{\nu}$$

$$x \boxed{L : L | \mathbb{N}}^0 = \mathbb{N}$$

$$x \boxed{L : L | \mathbb{N} : \mathbb{N}}^0_{\mu} = \underset{\mu}{L}^{-1\nu} \mathbb{N}$$

$$x \underline{L} : L = L \Rightarrow x \underline{\underset{\mu}{L}} : L^{\nu} = \underset{\mu}{L}^{\nu}$$

$$\Rightarrow \text{LHS} = \underset{\mu}{L}^{-1\nu} \overbrace{\left( \underbrace{x \boxed{L : L | \mathbb{N}}^0}_{=0} + \underbrace{x \boxed{L : L | \mathbb{N}}^0_{\nu} \mathbb{N}}_{=id} \right)} = \text{RHS}$$

$$x \times \mathbb{J} \xleftarrow{x \mathcal{L}} x \mathcal{L} \times \mathbb{R} \xleftarrow{x \mathcal{L}} \underbrace{x \mathcal{L} \times \mathbb{R}}_{= x \mathcal{L} \times \mathbb{R}} \xleftarrow{x \mathcal{L}} x \mathcal{L} \times \mathbb{R}$$

$$\begin{Bmatrix} x \\ \mathcal{L} \\ \mu \mathbb{N} \end{Bmatrix} \times \mathcal{L} = \begin{Bmatrix} x \mathcal{L} \\ x \mathcal{L}_{\mathbb{N}} \\ \underset{\mu}{L}^{-1\nu} x \left( \underbrace{\mathcal{L}_{\mathbb{N}}}_{\mathbb{N}} + \underbrace{\mathcal{L}_{\mathbb{N}}}_{\mathbb{N}} \right) \end{Bmatrix}$$

$$\left\{ \begin{array}{c} x \\ \mathbb{Z} \\ \mu \mathbb{N} \end{array} \right\} \times \underbrace{\mathbb{Z} : \mathbb{H}} \times \underbrace{\mathbb{H} : \mathbb{H}} = \left\{ \begin{array}{c} x \\ \mathbb{Z} \\ \mu \mathbb{N} \end{array} \right\} \times \underbrace{\mathbb{Z} \mathbb{H} : \mathbb{H} \mathbb{H}}$$

$$\text{LHS} = \left\{ \begin{array}{c} x \mathbb{Z} \\ x \mathbb{H} \mathbb{Z} \\ x \mathbb{Z}^{-1} \nu \mathbb{Z} \end{array} \right\} \underbrace{\left( \partial_\nu \mathbb{H} \mathbb{Z} + x \mathbb{H} \partial_\nu \mathbb{H} \right)} \times \underbrace{\mathbb{H} : \mathbb{H}} = \left\{ \begin{array}{c} x \mathbb{Z} \mathbb{H} \\ x \mathbb{Z} \mathbb{H} \mathbb{H} \\ x \mathbb{Z}^{-1} \nu \mathbb{Z} \end{array} \right\} \left( \partial_\nu \mathbb{H} \mathbb{Z} + x \mathbb{H} \partial_\nu \mathbb{H} \mathbb{Z}^{-1} \nu \mathbb{Z} \left( \partial_\lambda \mathbb{H} \mathbb{Z} + x \mathbb{H} \partial_\lambda \mathbb{H} \right) \right)$$

$$\text{RHS} = \left\{ \begin{array}{c} x \mathbb{Z} \mathbb{H} \\ x \mathbb{Z} \mathbb{H} \mathbb{H} \\ x \mathbb{Z}^{-1} \nu \mathbb{Z} \end{array} \right\} \underbrace{\left( \partial_\lambda \tilde{\mathbb{H}} \mathbb{H} \mathbb{Z} + x \tilde{\mathbb{H}} \mathbb{H} \partial_\lambda \mathbb{H} \right)}$$

$$= \left\{ \begin{array}{c} x \mathbb{Z} \mathbb{H} \\ x \mathbb{Z} \mathbb{H} \mathbb{H} \\ x \mathbb{Z}^{-1} \nu \mathbb{Z}^{-1} \lambda \end{array} \right\} \left( x \mathbb{Z}^{-1} \nu \mathbb{Z}^{-1} \lambda \partial_\lambda \mathbb{H} \mathbb{Z} + x \mathbb{Z} \mathbb{H} \partial_\lambda \mathbb{H} \mathbb{Z} + x \mathbb{Z} \mathbb{H} \partial_\lambda \mathbb{H} \mathbb{Z} \right)$$