$$\begin{cases} {}^{x}\mathfrak{s}^{\mu} = x^{\nu}\,{}_{\nu}\Lambda^{\mu} + \mathsf{L}^{\mu} = \tilde{x}^{\mu} \\ \underbrace{{}^{x}{}_{\sigma}\mathsf{h}}_{\mathsf{h}} = {}_{\sigma}\overset{-1}{\Lambda}{}^{\tau}\,{}_{\tau}\mathsf{h} = {}_{\sigma}\tilde{\mathsf{h}} \end{cases}$$

$${}_{\mu\sigma}\tilde{\mathsf{h}} = {}^{x}_{\mu}\overset{-1}{\mathfrak{s}}{}^{\nu}\ \overbrace{\underbrace{{}^{x}\partial_{\nu}\,{}_{\sigma}\mathsf{h}}_{\mathsf{h}} + \underbrace{{}^{x}{}_{\sigma}\mathsf{h}\partial^{\tau}}_{\mathsf{h}}\,{}_{\nu\tau}\mathsf{h}} = {}_{\mu}\overset{-1}{\Lambda}{}^{\nu}\ {}_{\sigma}\overset{-1}{\Lambda}{}^{\tau}\ {}_{\nu\tau}\mathsf{h}$$

$${}^{x}_{\mu}\mathfrak{s}^{\nu} = {}_{\mu}\Lambda^{\nu}$$

$$\underbrace{{}^{x}\partial_{\nu}\,{}_{\sigma}\mathsf{h}}_{\mathsf{h}} = 0$$

$$\underbrace{{}^{x}{}_{\sigma}\mathsf{h}\partial^{\tau}}_{\mathsf{h}} = {}_{\sigma}\Lambda^{\tau}$$

Poincare invariance ${}^{\tilde{x}}\mathcal{L}_{\tilde{\mathsf{h}}:\tilde{\mathsf{h}}} = {}^{x}\mathcal{L}_{\mathsf{h}:\mathsf{h}}$

$${}_{\mu\sigma}\tilde{\mathsf{h}} - {}_{\sigma\mu}\tilde{\mathsf{h}} = {}_{\mu}\overset{-1}{\Lambda}{}^{\lambda}\ {}_{\sigma}\overset{-1}{\Lambda}{}^{\varrho}\ {}_{\lambda\varrho}\mathsf{h} - {}_{\sigma}\overset{-1}{\Lambda}{}^{\lambda}\ {}_{\mu}\overset{-1}{\Lambda}{}^{\varrho}\ {}_{\lambda\varrho}\mathsf{h} = {}_{\mu}\overset{-1}{\Lambda}{}^{\lambda}\ {}_{\sigma}\overset{-1}{\Lambda}{}^{\varrho}\ {}_{\lambda\varrho}\mathsf{h} - {}_{\sigma}\overset{-1}{\Lambda}{}^{\varrho}\ {}_{\mu}\overset{-1}{\Lambda}{}^{\lambda}\ {}_{\zeta\lambda}\mathsf{h} = {}_{\mu}\overset{-1}{\Lambda}{}^{\lambda}\ {}_{\sigma}\overset{-1}{\Lambda}{}^{\varrho}\ \underbrace{{}_{\lambda\varrho}\mathsf{h} - {}_{\zeta\lambda}\mathsf{h}}$$

$$\text{LHS} = \overbrace{{}_{\mu\sigma}\tilde{\mathsf{h}} - {}_{\sigma\mu}\tilde{\mathsf{h}}}\,\eta^{\mu\nu}\,\eta^{\sigma\tau}\,\overbrace{{}_{\nu\tau}\tilde{\mathsf{h}} - {}_{\tau\nu}\tilde{\mathsf{h}}} = {}_{\mu}\overset{-1}{\Lambda}{}^{\lambda}\ {}_{\sigma}\overset{-1}{\Lambda}{}^{\varrho}\ \underbrace{{}_{\lambda\varrho}\mathsf{h} - {}_{\zeta\lambda}\mathsf{h}}\,\eta^{\mu\nu}\,\eta^{\sigma\tau}\ {}_{\nu}\overset{-1}{\Lambda}{}^{\varkappa}\ {}_{\tau}\overset{-1}{\Lambda}{}^{\vartheta}\ \underbrace{{}_{\varkappa\vartheta}\mathsf{h} - {}_{\vartheta\varkappa}\mathsf{h}}$$

$$= \underbrace{{}_{\lambda\varrho}\mathsf{h} - {}_{\zeta\lambda}\mathsf{h}}\ \underbrace{{}_{\mu}\overset{-1}{\Lambda}{}^{\lambda}\eta^{\mu\nu}\,{}_{\nu}\overset{-1}{\Lambda}{}^{\varkappa}}_{=\eta^{\lambda\varkappa}}\ \underbrace{{}_{\sigma}\overset{-1}{\Lambda}{}^{\varrho}\eta^{\sigma\tau}\,{}_{\tau}\overset{-1}{\Lambda}{}^{\vartheta}}_{=\eta^{\zeta\vartheta}}\ \underbrace{{}_{\varkappa\vartheta}\mathsf{h} - {}_{\vartheta\varkappa}\mathsf{h}} = \underbrace{{}_{\lambda\varrho}\mathsf{h} - {}_{\zeta\lambda}\mathsf{h}}\,\eta^{\lambda\varkappa}\,\eta^{\zeta\vartheta}\,\underbrace{{}_{\varkappa\vartheta}\mathsf{h} - {}_{\vartheta\varkappa}\mathsf{h}} = \text{RHS}$$

$$\begin{cases} {}^{x}\mathfrak{s} = x \\ \underbrace{{}^{x}{}_{\sigma}\mathsf{h}}_{\mathsf{h}} = {}_{\sigma}\mathsf{h} + {}^{x}_{\sigma\_}\mathsf{h}\,{}^{x}\mathsf{h}^{-1} = {}_{\sigma}\tilde{\mathsf{h}} \end{cases}$$

$${}_{\mu\sigma}\tilde{\mathsf{h}} = {}^{x}_{\mu\_}\overset{-1}{\mathfrak{s}}{}^{\nu}\ \overbrace{\underbrace{{}^{x}\partial_{\nu}\,{}_{\sigma}\mathsf{h}}_{\mathsf{h}} + \underbrace{{}^{x}{}_{\sigma}\mathsf{h}\partial^{\tau}}_{\mathsf{h}}\,{}_{\nu\tau}\mathsf{h}} = {}_{\mu\sigma}\mathsf{h} + {}^{x}_{\mu\sigma=}\mathsf{h}\,{}^{x}\mathsf{h}^{-1} - {}^{x}_{\sigma\_}\mathsf{h}\,{}^{x}\mathsf{h}^{-1}\,{}^{x}_{\mu\_}\mathsf{h}\,{}^{x}\mathsf{h}^{-1}$$

$$\underbrace{{}^{x}\partial_{\nu}\,{}_{\sigma}\mathsf{h}}_{\mathsf{h}} = \underline{{}^{x}_{\nu}{}_{\sigma\_}\mathsf{h}\mathsf{h}^{-1}} = {}^{x}_{\nu\sigma=}\mathsf{h}\,{}^{x}\mathsf{h}^{-1} + {}^{x}_{\sigma\_}\mathsf{h}\,\underline{{}^{x}_{\nu}\mathsf{h}^{-1}} = {}^{x}_{\nu\sigma=}\mathsf{h}\,{}^{x}\mathsf{h}^{-1} - {}^{x}_{\sigma\_}\mathsf{h}\,{}^{x}\mathsf{h}^{-1}\,{}^{x}_{\nu\_}\mathsf{h}\,{}^{x}\mathsf{h}^{-1}$$

$$\underbrace{{}^{x}{}_{\sigma}\mathsf{h}\partial^{\tau}}_{\mathsf{h}} = \frac{\partial\,{}_{\sigma}\mathsf{h}}{\partial\,{}_{\tau}\mathsf{h}} = {}_{\sigma}\delta^{\tau}$$

$$\underbrace{{}^{x}{}_{\sigma}\mathsf{h}\partial^{\tau}}_{\mathsf{h}}\ {}_{\nu\tau}\mathsf{h} = {}_{\sigma}\delta^{\tau}\ {}_{\nu\tau}\mathsf{h} = {}_{\nu\sigma}\mathsf{h}$$

$$\text{LHS} = {}_{\mu}\delta^{\nu}\ \overbrace{{}^{x}_{\nu\sigma=}\mathsf{h}\,{}^{x}\mathsf{h}^{-1} - {}^{x}_{\sigma\_}\mathsf{h}\,{}^{x}\mathsf{h}^{-1}\,{}^{x}_{\nu\_}\mathsf{h}\,{}^{x}\mathsf{h}^{-1} + {}_{\nu\sigma}\mathsf{h}} = \text{RHS}$$

$$\text{gauge invariance } {}^{\tilde{x}}\mathcal{L}_{\tilde{\mathsf{A}}:\tilde{\mathsf{A}}} = {}^{x}\mathcal{L}_{\mathsf{A}:\mathsf{A}}$$

$${}_{\mu\sigma}\tilde{\mathsf{A}} - {}_{\sigma\mu}\tilde{\mathsf{A}} = \overbrace{{}_{\mu\sigma}\mathsf{A} + {}^{x}_{\mu\sigma\underline{=}}\mathsf{Y}\,{}^{x}\mathsf{Y}^{-1} - {}^{x}_{\sigma\underline{\ }}\mathsf{Y}\,{}^{x}\mathsf{Y}^{-1}\,{}^{x}_{\mu\underline{\ }}\mathsf{Y}\,{}^{x}\mathsf{Y}^{-1}} - \overbrace{{}_{\sigma\mu}\mathsf{A} + {}^{x}_{\sigma\mu\underline{=}}\mathsf{Y}\,{}^{x}\mathsf{Y}^{-1} - {}^{x}_{\mu\underline{\ }}\mathsf{Y}\,{}^{x}\mathsf{Y}^{-1}\,{}^{x}_{\sigma\underline{\ }}\mathsf{Y}\,{}^{x}\mathsf{Y}^{-1}} = {}_{\mu\sigma}\mathsf{A} - {}_{\sigma\mu}\mathsf{A}$$

$$\text{LHS} = \overbrace{{}_{\mu\sigma}\tilde{\mathsf{A}} - {}_{\sigma\mu}\tilde{\mathsf{A}}}\,\eta^{\mu\nu}\,\eta^{\sigma\tau}\,\overbrace{{}_{\nu\tau}\tilde{\mathsf{A}} - {}_{\tau\nu}\tilde{\mathsf{A}}} = \overbrace{{}_{\mu\sigma}\mathsf{A} - {}_{\sigma\mu}\mathsf{A}}\,\eta^{\mu\nu}\,\eta^{\sigma\tau}\,\overbrace{{}_{\nu\tau}\mathsf{A} - {}_{\tau\nu}\mathsf{A}} = \text{RHS}$$