

electric stability ${}^x \mathcal{L}_{\mathfrak{N}; \mu \mathfrak{N}} = {}^{x\mathfrak{N}} \mathcal{L}_{\mathfrak{N}; \mu \mathfrak{N}} \overbrace{\det {}^{x\mathfrak{N}} \underline{\mathfrak{N}}}^{\overbrace{\partial_{\nu \mathfrak{N}} + {}^x \mathfrak{N} \partial_{\nu \tau \mathfrak{N}}}}$

$$x^\nu : {}^x \mathfrak{N}; \mu \mathfrak{N} \in \mathbb{R}^d \times {}^N \mathbb{R} \times {}^N_d \mathbb{R} \xrightarrow[\text{current}]{\mathcal{J}^\mu} \mathbb{R} \ni {}^x \mathcal{J}_{\mathfrak{N}; \mu \mathfrak{N}}^\mu = {}^x \mathcal{J}_{\mathfrak{N}; \mu \mathfrak{N}}^\mu$$

$${}^x \mathcal{J}_{\mathfrak{N}; \mu \mathfrak{N}}^\mu = \mathfrak{b}_{x \nu}^\nu \overbrace{\delta^{\mu x} \mathcal{L}_{\mathfrak{N}; \mathfrak{N}} - \mathfrak{N}_{\nu \tau} \mathcal{L}^\mu \partial^\sigma}_{\mathfrak{N}; \mathfrak{N}} + \mathcal{L}^\mu \partial^\sigma \mathfrak{N}_{\mathfrak{N}} = \mathfrak{b}_x^\mu \mathcal{L}_{\mathfrak{N}; \mathfrak{N}} - \mathcal{L}^\mu \partial^\sigma \underbrace{\mathfrak{b}_{x \nu \tau}^\nu \mathfrak{N} + \mathfrak{N}_{\mathfrak{N}}}_{\mathfrak{N}; \mathfrak{N}}$$

$$\mu \mathfrak{J}_{\mathfrak{N}}^\mu = \partial_\mu \mathcal{J}_{\mathfrak{N}}^\mu = 0 \text{ conserved current}$$

$${}^x \mathcal{L}_{\mathfrak{N}; \mu \mathfrak{N}} = {}^{x\mathfrak{N}_\varepsilon} \mathcal{L}_{\mathfrak{N}; \mu \mathfrak{N}} \overbrace{\det {}^{x\mathfrak{N}_\varepsilon} \underline{\mathfrak{N}}}^{\overbrace{\partial_{\nu \varepsilon \mathfrak{N}} + {}^x \mathfrak{N}_\varepsilon \partial_{\nu \tau \mathfrak{N}_\varepsilon}}}$$

$$0 = \partial_\varepsilon^0 {}^x \mathcal{L}_{\mathfrak{N}; \mu \mathfrak{N}} = \partial_\varepsilon^0 {}^{x\mathfrak{N}_\varepsilon} \mathcal{L}_{\mathfrak{N}; \mu \mathfrak{N}} \overbrace{\det {}^{x\mathfrak{N}_\varepsilon} \underline{\mathfrak{N}}}^{\overbrace{\partial_{\nu \varepsilon \mathfrak{N}} + {}^x \mathfrak{N}_\varepsilon \partial_{\nu \tau \mathfrak{N}_\varepsilon}}} = \partial_\varepsilon^0 {}^{x\mathfrak{N}_\varepsilon} \mathcal{L}_{\mathfrak{N}; \mu \mathfrak{N}} \overbrace{\det {}^{x\mathfrak{N}_\varepsilon} \underline{\mathfrak{N}}}^{\overbrace{\partial_{\nu \varepsilon \mathfrak{N}} + {}^x \mathfrak{N}_\varepsilon \partial_{\nu \tau \mathfrak{N}_\varepsilon}}} + {}^x \mathcal{L}_{\mathfrak{N}; \mu \mathfrak{N}} \underbrace{\partial_\varepsilon^0 \det {}^{x\mathfrak{N}_\varepsilon} \underline{\mathfrak{N}}}_{= \text{tr } \mathfrak{b}_x}$$

$$= {}^x \mathfrak{b}^\nu \mathcal{L}_{\mathfrak{N}; \mathfrak{N}} + \mathcal{L}^\sigma \mathfrak{N}_{\mathfrak{N}} + \mathcal{L}^\mu \partial^\sigma \overbrace{\mathfrak{N}_{\mathfrak{N}} + \mathfrak{N}_{\mathfrak{N}} - \mathfrak{b}_{\nu \tau}^\nu \mathfrak{N} + \mathfrak{N}_{\mathfrak{N}}}}^{\mathfrak{N}_{\mathfrak{N}}}$$

$$= {}^x \mathfrak{b}^\nu \mathcal{L}_{\mathfrak{N}; \mathfrak{N}} + \mathcal{L}^\sigma \mathfrak{N}_{\mathfrak{N}} + \mathcal{L}^\mu \partial^\sigma \overbrace{\mathfrak{N}_{\mathfrak{N}} + \mathfrak{N}_{\mathfrak{N}} + \mathfrak{b}_{\nu}^\nu \delta^{\mu x} \mathcal{L}_{\mathfrak{N}; \mathfrak{N}} - \mathfrak{N}_{\mathfrak{N}} \mathcal{L}^\mu \partial^\sigma}_{\mathfrak{N}; \mathfrak{N}}$$

$$\text{fields } 0 = {}^x \mathfrak{b}^\nu \mathcal{L}_{\mathfrak{N}; \mathfrak{N}} + \mathcal{L}^\sigma \mathfrak{N}_{\mathfrak{N}} + \mathcal{L}^\mu \partial^\sigma \overbrace{\mathfrak{N}_{\mathfrak{N}} + \mathfrak{N}_{\mathfrak{N}}}}^{= \mathfrak{N}_{\mathfrak{N}}} + \mathfrak{b}_{\mu}^\nu \overbrace{\delta^{\mu x} \mathcal{L}_{\mathfrak{N}; \mathfrak{N}} - \mathfrak{N}_{\mathfrak{N}} \mathcal{L}^\mu \partial^\sigma}_{\mathfrak{N}; \mathfrak{N}}$$

$$\stackrel{\text{harmonic}}{=} \underbrace{{}^x \mathfrak{b}^\nu \delta^{\mu x} \mathcal{L}_{\mathfrak{N}; \mathfrak{N}} - \mathfrak{N}_{\mathfrak{N}} \mathcal{L}^\mu \partial^\sigma}_{\mu} + \underbrace{\mathcal{L}^\mu \partial^\sigma \mathfrak{N}_{\mathfrak{N}}}_{\mu} + \mathcal{L}^\mu \partial^\sigma \mathfrak{N}_{\mathfrak{N}} + \mathfrak{b}_{\mu}^\nu \overbrace{\delta^{\mu x} \mathcal{L}_{\mathfrak{N}; \mathfrak{N}} - \mathfrak{N}_{\mathfrak{N}} \mathcal{L}^\mu \partial^\sigma}_{\mathfrak{N}; \mathfrak{N}}$$

$$= \underbrace{{}^x \mathfrak{b}^\nu \delta^{\mu x} \mathcal{L}_{\mathfrak{N}; \mathfrak{N}} - \mathfrak{N}_{\mathfrak{N}} \mathcal{L}^\mu \partial^\sigma}_{\mu} + \mathcal{L}^\mu \partial^\sigma \mathfrak{N}_{\mathfrak{N}}$$

conserved electric charge $\partial_t \int_S^{\text{ds}} \mathcal{J}^0 = 0$

$$0 = \partial_\mu \mathcal{J}^\mu = \mathfrak{d} \cdot \bar{\mathcal{J}} + \partial_t \mathcal{J}^0$$

$$\Rightarrow 0 = \int_S^{\text{ds}} \partial_\mu \mathcal{J}^\mu = \int_S^{\text{ds}} \frac{\mathfrak{d} \cdot \bar{\mathcal{J}}}{2} (= 0) + \int_S^{\text{ds}} \partial_t \mathcal{J}^0 = \partial_t \int_S^{\text{ds}} \mathcal{J}^0$$

energy-momentum tensor

$$\begin{aligned} \lambda \{A\}^\nu &= \partial_\lambda A_\mu \partial^{\mu\nu} \{A\} - \lambda \delta^\nu \{A\} = \lambda \partial_\mu A \frac{\partial \{A\}}{\partial_\mu A_\nu} - \lambda \delta^\nu \{A\} \\ &= \partial_\lambda A_\mu \eta^{\mu\zeta} \eta^{\nu\lambda} \underbrace{\partial_\zeta A_\lambda - \partial_\lambda A_\zeta} + \frac{\lambda \delta^\nu}{4} \underbrace{\partial_\rho A_\sigma - \partial_\sigma A_\rho}_{\text{old}} \eta^{\zeta\kappa} \eta^{\sigma\lambda} \underbrace{\partial_\zeta A_\lambda - \partial_\lambda A_\zeta} \end{aligned}$$

$$\overbrace{\begin{pmatrix} x \\ \sigma \mathbb{N} \\ \mu \sigma \mathbb{N} \end{pmatrix} \times \underbrace{\mathcal{N}} \times \underbrace{\mathcal{H}}}_{\text{}} = \begin{pmatrix} x \\ \sigma \mathbb{N} \\ \mu \sigma \mathbb{N} \end{pmatrix} \times \underbrace{\mathcal{N} \mathcal{H}}_{\text{}}$$

$$\text{LHS} = \begin{pmatrix} x \mathcal{N} \\ x \mathcal{H} \mathbb{N} \\ x \mathcal{N}^{-1} \nu \overbrace{\left(\partial_\nu \mathcal{H} + \mathcal{H} \partial_{\nu \tau} \mathbb{N} \right)} \end{pmatrix} \times \underbrace{\mathcal{H}} = \begin{pmatrix} x \mathcal{N} \mathcal{H} \\ x \mathcal{N} \mathcal{H} x \mathcal{H} \mathbb{N} \\ x \mathcal{N}^{-1} \nu \left(x \mathcal{N} \partial_\nu \mathcal{H} + x \mathcal{N} \mathcal{H} \partial_{x \mathcal{H} \mathbb{N}} \overbrace{\left(\partial_\lambda \mathcal{H} + \mathcal{H} \partial_{\lambda \epsilon} \mathbb{N} \right)} \right) \end{pmatrix}$$

$$\text{RHS} = \begin{pmatrix} x \mathcal{N} \mathcal{H} \\ x \mathcal{N} \mathcal{H} x \mathcal{H} \mathbb{N} \\ x \mathcal{N}^{-1} \lambda \overbrace{\left(\partial_\lambda \tilde{\mathcal{H}} + \tilde{\mathcal{H}} \partial_{\lambda \epsilon} \mathbb{N} \right)} \end{pmatrix}$$

$$= \begin{pmatrix} x \mathcal{N} \mathcal{H} \\ x \mathcal{N} \mathcal{H} x \mathcal{H} \mathbb{N} \\ x \mathcal{N}^{-1} \nu x \mathcal{N}^{-1} \lambda \left(x \mathcal{N} \partial_\nu \mathcal{H} + x \mathcal{N} \mathcal{H} \partial_{x \mathcal{H} \mathbb{N}} \partial_\lambda \mathcal{H} + x \mathcal{N} \mathcal{H} \partial_{x \mathcal{H} \mathbb{N}} \mathcal{H} \partial_{\lambda \epsilon} \mathbb{N} \right) \end{pmatrix}$$