$$\text{Pauli matrices } \sigma^0 = \begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array} = \sigma_0 : \sigma^1 = \begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array} = -\sigma_1 : \sigma^2 = \begin{array}{c|c} 0 & -i \\ \hline i & 0 \end{array} = -\sigma_2 : \sigma^3 = \begin{array}{c|c} 1 & 0 \\ \hline 0 & -1 \end{array} = -\sigma_3$$

$$\gamma^\mu = \begin{array}{c|c} 0 & \sigma^\mu \\ \hline \sigma^2 \overset{t}{\sigma}{}^\mu \sigma^2 & 0 \end{array} = \begin{array}{c|c} 0 & \sigma^\mu \\ \hline \widetilde{\sigma}^\mu & 0 \end{array}$$

$$\begin{cases} \sigma^2 \mathcal{J} \sigma^2 = \overset{-T}{\mathcal{J}} \\ \sigma^2 \bar{\mathcal{J}} \sigma^2 = \overset{-*}{\mathcal{J}} \end{cases}$$

$$\sigma^2 \begin{array}{c|c} a & b \\ \hline c & d \end{array} \sigma^2 = \begin{array}{c|c} 0 & -i \\ \hline i & 0 \end{array} \begin{array}{c|c} a & b \\ \hline c & d \end{array} \begin{array}{c|c} 0 & -i \\ \hline i & 0 \end{array} = \begin{array}{c|c} d & -c \\ \hline -b & a \end{array} = \overset{-T}{\mathcal{J}}$$

$$\text{left Weyl spinors } \frac{1}{2}{:}0 \begin{cases} {}^2\mathbb{C} \ni \mathcal{H} = \left(\overset{A}{\mathcal{H}}\right) = \psi_L = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \left(\psi_A\right) & \overbrace{\mathcal{J} \ltimes \mathcal{H}}^{A} = {}^A\mathcal{J}_B \overset{B}{\mathcal{H}} \\ \mathbb{C}_2 \ni \mathcal{Y} = \left(\mathcal{Y}_A\right) = \psi^*_R = \begin{bmatrix} \psi^1 & \psi^2 \end{bmatrix} = \left(\psi^A\right) & \underbrace{\mathcal{J} \ltimes \mathcal{Y}}_{B} = \mathcal{Y}_A \, {}^{A}\overset{-1}{\mathcal{J}}_B \end{cases}$$

$$\text{right Weyl spinors } 0{:}\frac{1}{2} \begin{cases} {}^2\mathbb{C} \ni \overset{*}{\mathcal{Y}} = \left(\overset{*}{\mathcal{Y}}_A\right) = \psi_R = \begin{bmatrix} \bar{\psi}^{\dot{1}} \\ \bar{\psi}^{\dot{2}} \end{bmatrix} = \left(\bar{\psi}^{\dot{A}}\right) & \underbrace{\mathcal{J} \overset{*}{\ltimes} \mathcal{Y}}_{B} = {}^{A}\overset{-*}{\mathcal{J}}_B \overset{*}{\mathcal{Y}}_A \\ \mathbb{C}_2 \ni \overset{*}{\mathcal{H}} = \left(\underset{*}{\overset{A}{\mathcal{H}}}\right) = \psi^*_L = \begin{bmatrix} \bar{\psi}_{\dot{1}} & \bar{\psi}_{\dot{2}} \end{bmatrix} = \left(\bar{\psi}_{\dot{A}}\right) & \overbrace{\mathcal{J} \underset{*}{\ltimes} \mathcal{H}}^{A} = \underset{*}{\overset{B}{\mathcal{H}}} \, {}^A\mathcal{J}_{*B} \end{cases}$$

$$\text{Dirac } \frac{1}{2}{:}0 \oplus 0{:}\frac{1}{2} \begin{cases} \begin{bmatrix} \mathcal{H} \\ \overset{*}{\mathcal{Y}} \end{bmatrix} = \begin{bmatrix} \overset{A}{\mathcal{H}} \\ \bar{\mathcal{Y}}_A \end{bmatrix} = \psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \bar{\psi}^{\dot{1}} \\ \bar{\psi}^{\dot{2}} \end{bmatrix} = \begin{bmatrix} \psi_A \\ \bar{\psi}^{\dot{A}} \end{bmatrix} \\ \begin{bmatrix} \mathcal{H} \\ \overset{*}{\mathcal{Y}} \end{bmatrix}^\sharp = \begin{bmatrix} \mathcal{H} \\ \overset{*}{\mathcal{Y}} \end{bmatrix}^* \begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array} = \begin{bmatrix} \mathcal{Y} & \overset{*}{\mathcal{H}} \end{bmatrix} = \begin{bmatrix} \mathcal{Y}_A & \overline{\overset{A}{\mathcal{H}}} \end{bmatrix} = \overset{\sharp}{\psi} = \overset{*}{\psi}\gamma_0 = \begin{bmatrix} \overset{*}{\psi}_R & \overset{*}{\psi}_L \end{bmatrix} = \left(\psi^{\dot{1}} \psi^{\dot{2}} \bar{\psi}_1 \bar{\psi}_2\right) = \begin{bmatrix} \psi^{\dot{A}} & \bar{\psi}_A \end{bmatrix} \end{cases}$$

$$\text{Majorana spinors } \frac{1}{2}{:}0 \underset{\mathbb{R}}{\oplus} 0{:}\frac{1}{2} \begin{cases} \begin{bmatrix} \bar{\psi}^{\dot{1}} \\ \bar{\psi}^{\dot{2}} \end{bmatrix} = \Psi_R = -i\sigma^2 \bar{\Psi}_L = \begin{array}{c|c} 0 & -1 \\ \hline 1 & 0 \end{array} \begin{bmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{bmatrix} = \begin{bmatrix} -\bar{\psi}_2 \\ \bar{\psi}_1 \end{bmatrix} \\ \begin{bmatrix} \Psi_L \\ -i\sigma^2 \bar{\Psi}_L \end{bmatrix} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ -\bar{\psi}_2 \\ \bar{\psi}_1 \end{bmatrix} = \begin{bmatrix} \overset{A}{\mathcal{H}} \\ -i^B \sigma^2_A \overline{\overset{A}{\mathcal{H}}} \end{bmatrix} \end{cases}$$

$$2^{N} \ni A \subset N$$

$$\text{dof } \begin{bmatrix} \Psi^{A} \\ \bar{\Psi}_{A} \end{bmatrix} : \begin{bmatrix} {}_{\mu}\Psi^{A} \\ {}_{\mu}\bar{\Psi}_{A} \end{bmatrix} \in {}^{2^{N}}\mathbb{K} \times {}^{2^{N}}_{d}\mathbb{K}$$

$$\boxed{\begin{matrix} \Psi & & \Psi \\ \overset{*}{\Psi} & : & \overset{*}{\Psi} \end{matrix}} = \boxed{\begin{matrix} \Psi^{A} & & {}_{\mu}\Psi^{A} \\ \overset{*}{\Psi}_{A} & : & {}_{\mu}\overset{*}{\Psi}_{A} \end{matrix}} = \begin{bmatrix} \Psi \\ \overset{*}{\Psi} \end{bmatrix}^{\sharp} \underbrace{\gamma^{m} \begin{bmatrix} {}_{\mu}\Psi \\ {}_{\mu}\overset{*}{\Psi} \end{bmatrix} - m \begin{bmatrix} \Psi \\ \overset{*}{\Psi} \end{bmatrix}} = \begin{bmatrix} \Psi & \underset{*}{\Psi} \end{bmatrix} \underbrace{\begin{array}{c|c} 0 & \sigma^{\mu} \\ \hline \widetilde{\sigma}^{\mu} & 0 \end{array} \begin{bmatrix} {}_{\mu}\Psi \\ {}_{\mu}\overset{*}{\Psi} \end{bmatrix} - m \begin{bmatrix} \Psi \\ \overset{*}{\Psi} \end{bmatrix}}$$

$$= \Psi \sigma^{\mu} \, {}_{\mu}\overset{*}{\Psi} + \underset{*}{\Psi} \, \widetilde{\sigma}^{\mu} \, {}_{\mu}\Psi - m \left(\Psi\Psi + \underset{*}{\Psi}\overset{*}{\Psi} \right) = \Psi_{A} \, {}^{A}\sigma^{\mu}_{B} \, {}_{\mu}\overset{*}{\Psi}_{B} + \underset{*}{\Psi}^{A} \, {}^{A}\widetilde{\sigma}^{\mu}_{B} \, {}_{\mu}\Psi^{B} - m \left(\Psi_{A}\Psi^{A} + \underset{*}{\Psi}^{A}\overset{*}{\Psi}_{A} \right)$$

$$= \Psi_{A} \left({}^{A}\sigma^{\mu}_{B} \, {}_{\mu}\overset{*}{\Psi}_{B} - m\Psi^{A} \right) + \underset{*}{\Psi}^{A} \left({}^{A}\widetilde{\sigma}^{\mu}_{B} \, {}_{\mu}\Psi^{B} - m\overset{*}{\Psi}_{A} \right)$$

$$= \overset{\sharp}{\psi} \, \gamma^{\mu} \, {}_{\mu}\Psi - m \overset{\sharp}{\psi} \psi = \psi^{*}_{R} \, \sigma^{\mu} \, {}_{\mu}\Psi_{R} + \psi^{*}_{L} \, \sigma^{2} \, \overset{t}{\sigma}^{\mu} \, \sigma^{2} \, {}_{\mu}\Psi_{L} - m \left(\psi^{*}_{R}\psi_{L} + \psi^{*}_{L}\psi_{R} \right)$$

$$= \psi^{A} \, \sigma^{\mu}_{A\dot{B}} \, {}_{\mu}\overset{*}{\Psi}^{\dot{B}} + \overset{*}{\psi}_{\dot{A}} \, \overbrace{\sigma^{2}\overset{t}{\sigma}^{\mu}\sigma^{2}}^{\dot{A}B} \, {}_{\mu}\Psi_{B} - m \left(\psi^{A}\psi_{A} + \overset{*}{\psi}_{\dot{A}}\overset{*}{\psi}^{\dot{A}} \right)$$

$$^{x}\boxed{\begin{matrix} \Psi & \\ \overset{*}{\Psi} & : \end{matrix}} = {}^{x}\boxed{\begin{matrix} \Psi^{A} & \\ \bar{\Psi}_{A} & : \end{matrix}} = \boxed{\begin{matrix} {}^{x}\Psi & & {}^{x}\Psi \\ {}^{x}\overset{*}{\Psi} & : & {}^{x}\overset{*}{\underset{-}{\Psi}} \end{matrix}} = \boxed{\begin{matrix} {}^{x}\Psi^{A} & & {}^{x}_{\mu}\Psi^{A}_{-} \\ {}^{x}\bar{\Psi}_{A} & : & {}^{x}_{\mu}\bar{\Psi}_{-A} \end{matrix}} = \begin{bmatrix} {}^{x}\Psi \\ {}^{x}\overset{*}{\Psi} \end{bmatrix}^{\sharp} \gamma^{\mu} \begin{bmatrix} {}^{x}_{\mu}\Psi_{-} \\ {}^{x}_{\mu}\overset{*}{\Psi}_{-} \end{bmatrix} - m \begin{bmatrix} {}^{x}\Psi \\ {}^{x}\overset{*}{\Psi} \end{bmatrix}^{\sharp} \begin{bmatrix} {}^{x}\Psi \\ {}^{x}\overset{*}{\Psi} \end{bmatrix}$$

$$= \begin{bmatrix} {}^{x}\Psi & {}^{x}\underset{*}{\Psi} \end{bmatrix} \begin{array}{c|c} 0 & \sigma^{\mu} \\ \hline \widetilde{\sigma}^{\mu} & 0 \end{array} \begin{bmatrix} {}^{x}_{\mu}\Psi_{-} \\ {}^{x}_{\mu}\overset{*}{\Psi}_{-} \end{bmatrix} - m \begin{bmatrix} {}^{x}\Psi & {}^{x}\underset{*}{\Psi} \end{bmatrix} \begin{bmatrix} {}^{x}\Psi \\ {}^{x}\overset{*}{\Psi} \end{bmatrix} = {}^{x}\Psi \, \sigma^{\mu} \, {}^{x}_{\mu}\overset{*}{\Psi}_{-} + {}^{x}\underset{*}{\Psi} \, \widetilde{\sigma}^{\mu} \, {}^{x}_{\mu}\Psi_{-} - m \left({}^{x}\Psi {}^{x}\Psi + {}^{x}\underset{*}{\Psi} {}^{x}\overset{*}{\Psi} \right)$$

$$= {}^{x}\Psi_{A} \, {}^{A}\sigma^{\mu}_{B} \, {}^{x}_{\mu}\bar{\Psi}_{-B} + {}^{x}\underset{*}{\Psi}^{A} \, {}^{A}\widetilde{\sigma}^{\mu}_{B} \, {}^{x}_{\mu}\Psi^{B}_{-} - m \left({}^{x}\Psi_{A} {}^{x}\Psi^{A} + {}^{x}\underset{*}{\Psi}^{A} {}^{x}\bar{\Psi}_{A} \right) = {}^{x}\Psi_{A} \left({}^{A}\sigma^{\mu}_{B} \, {}^{x}_{\mu}\bar{\Psi}_{-B} - m^{x}\Psi^{A} \right) + {}^{x}\underset{*}{\Psi}^{A} \left({}^{A}\widetilde{\sigma}^{\mu}_{B} \, {}^{x}_{\mu}\Psi^{B}_{-} - m^{x}\bar{\Psi}_{A} \right)$$

$$\text{field Lagrangian } \overset{\sharp}{\psi} \, \gamma^{\mu} \, \partial_{\mu}\psi - m \overset{\sharp}{\psi} \psi = \psi^{*}_{R} \, \sigma^{\mu} \, \partial_{\mu} \, \psi_{R} + \psi^{*}_{L} \, \sigma^{2} \, \overset{t}{\sigma}^{\mu} \, \sigma^{2} \, \partial_{\mu} \, \psi_{L} - m \left(\psi^{*}_{R}\psi_{L} + \psi^{*}_{L}\psi_{R} \right)$$

$$= \psi^{A} \, \sigma^{\mu}_{A\dot{B}} \, \partial_{\mu} \, \bar{\psi}^{\dot{B}} + \bar{\psi}_{\dot{A}} \, \overbrace{\sigma^{2}\overset{t}{\sigma}^{\mu}\sigma^{2}}^{\dot{A}B} \, \partial_{\mu} \, \psi_{B} - m \left(\psi^{A}\psi_{A} + \bar{\psi}_{\dot{A}}\bar{\psi}^{\dot{A}} \right)$$

$$\overset{\sharp}{\psi} \psi = \psi^{*}_{R} \, \psi_{L} + \psi^{*}_{L} \, \psi_{R} = \psi^{A} \, \psi_{A} + \bar{\psi}_{\dot{A}} \, \bar{\psi}^{\dot{A}}$$

$$\overset{\sharp}{\psi} \, \gamma^{\mu} \, \partial_{\mu}\psi = \psi^{*}_{R} \, \sigma^{\mu} \, \partial_{\mu} \, \psi_{R} + \psi^{*}_{L} \, \sigma^{2} \, \overset{t}{\sigma}^{\mu} \, \sigma^{2} \, \partial_{\mu} \, \psi_{L} = \psi^{A} \, \underbrace{\sigma^{\mu}}_{A\dot{A}} \, \partial_{\mu} \, \bar{\psi}^{\dot{A}} + \bar{\psi}_{\dot{A}} \, \overbrace{\sigma^{2}\overset{t}{\sigma}^{\mu}\sigma^{2}}^{\dot{A}A} \, \partial_{\mu} \, \psi_{A}$$