

$$\begin{aligned}
D &= \gamma^\mu \partial_\mu \\
\mathcal{D} &= \dot{\gamma}^\mu \partial_\mu \\
\kappa = 1 &\Rightarrow D = \mathcal{D} \\
\Gamma D &= -D\Gamma \\
\Gamma \mathcal{D} &= -\mathcal{D}\Gamma \\
\Gamma \gamma^\mu &= -\gamma^\mu \Gamma \Rightarrow \Gamma \underbrace{\gamma^\mu \partial}_\mu = -\underbrace{\gamma^\mu \partial}_\mu \Gamma \\
\dot{\gamma}^\mu = \pm \gamma^\mu &\Rightarrow \Gamma \dot{\gamma}^\mu = -\dot{\gamma}^\mu \Gamma \Rightarrow \Gamma \underbrace{\dot{\gamma}^\mu \partial}_\mu = -\underbrace{\dot{\gamma}^\mu \partial}_\mu \Gamma
\end{aligned}$$

$$\kappa = i \Rightarrow D \text{ skew-adj}$$

$$\kappa = 1 \Rightarrow \gamma^0 D \text{ skew-adj } \gamma^0 \mathcal{D}$$

$$\kappa = i: \overline{\gamma^\mu \partial}^* = \partial_\mu \dot{\gamma}^\mu = -\partial_\mu \gamma^\mu = -\gamma^\mu \partial_\mu \Leftarrow \gamma^\mu \text{ cst}$$

$$\kappa = 1: \overline{\gamma^0 \gamma^k}^* = \dot{\gamma}^k \dot{\gamma}^0 = -\gamma^k \gamma^0 = \gamma^0 \gamma^k$$

$$\overline{\gamma^0 \gamma^0}^* = \dot{\gamma}^0 \dot{\gamma}^0 = \gamma^0 \gamma^0 \Rightarrow$$

$$\overline{\gamma^0 \gamma^\mu \partial}^* = \partial_\mu \dot{\gamma}^0 \overline{\gamma^0 \gamma^\mu}^* = -\partial_\mu \gamma^0 \gamma^\mu = -\underbrace{\gamma^0 \gamma^\mu}_\mu \partial \Leftarrow \gamma^0 \gamma^\mu \text{ cst}$$

$$\overline{\gamma^0 \dot{\gamma}^\mu \partial}^* = \partial_\mu \dot{\gamma}^0 \overline{\gamma^0 \dot{\gamma}^\mu}^* = -\partial_\mu \gamma^0 \dot{\gamma}^\mu = -\underbrace{\gamma^0 \dot{\gamma}^\mu}_\mu \partial \Leftarrow \gamma^0 \dot{\gamma}^\mu \text{ cst}$$

$$D^* = -D$$

$$\mathcal{D}^* = -\mathcal{D}$$

$$\underbrace{\gamma^0 \bar{\gamma}^2}_\mathcal{D} = -D \underbrace{\gamma^0 \bar{\gamma}^0}_\mathcal{D}$$

$$\tilde{\mathcal{D}} = -\tilde{D}$$

$$\tilde{\mathcal{D}} = \tilde{\gamma}^\mu \partial_\mu = -\gamma^\mu \tilde{\partial}_\mu = -\gamma^\mu \partial_\mu \tilde{} = -\tilde{D}$$

$$\underbrace{\gamma^0 \bar{\gamma}^2}_\mathcal{D} \mathcal{D} \underbrace{\gamma^0 \bar{\gamma}^2}_\mathcal{D} = D$$

$$\tilde{\mathcal{D}} \tilde{} = D$$

$$\tilde{D} \tilde{} = \mathcal{D}$$

$$\tilde{\mathcal{D}} \tilde{} = -D \tilde{} = D$$

$$\tilde{D} \tilde{} = -\tilde{} \mathcal{D} = \mathcal{D}$$