

$$x \boxed{\begin{array}{c} A \\ \Gamma : L \\ \Psi_B^* \end{array}}_A = \begin{bmatrix} A & B \\ \Gamma_B & \Psi_B^* \\ \Psi_B^* & \Gamma_{-A} \end{bmatrix}$$

$$x \underline{\Gamma : L}^\nu = x^\mu \mu \Gamma^\nu + L^\nu$$

$$\left\{ \begin{array}{l} \Gamma \sigma^\nu \Gamma^* = \sigma^\mu \mu \Gamma^\nu \quad A \Gamma_C^C \sigma_D^\nu \Gamma_D^B = A \sigma_{B\mu}^\mu \Gamma^\nu \\ \Gamma^* \tilde{\sigma}^\nu \Gamma^* = \mu \Gamma^\nu \tilde{\sigma}^\mu \quad C \Gamma_{-A}^1 C \tilde{\sigma}_D^\nu D \Gamma_B^1 = \mu \Gamma^\nu A \tilde{\sigma}_B^\mu \end{array} \right.$$

$$\sigma^2 \text{ LHS } \sigma^2 = \underbrace{\sigma^2 \Gamma^* \sigma^2}_{\tilde{\sigma}^\nu} \underbrace{\tilde{\sigma}^\nu \sigma^2 \Gamma^* \sigma^2}_{\tilde{\sigma}^\nu} = \tilde{\sigma}^\nu \tilde{\sigma}^\nu \tilde{\sigma}^\nu = \overbrace{\Gamma \sigma^\nu \Gamma^*}^{\tilde{\sigma}^\nu} = \overbrace{\sigma^\mu \mu \Gamma^\nu}^{\tilde{\sigma}^\nu} = \mu \Gamma^\nu \tilde{\sigma}^\mu = \sigma^2 \text{ RHS } \sigma^2$$

$$\underline{\Gamma \mathcal{F}} \sigma^\lambda \overline{\Gamma \mathcal{F}}^* = \underline{\Gamma \mathcal{F} \sigma^\lambda \mathcal{F}^*} \Gamma^* = \underline{\Gamma \sigma^\nu \nu \mathcal{F}^\lambda} \Gamma^* = \underline{\Gamma \sigma^\nu \Gamma^*} \nu \mathcal{F}^\lambda = \sigma^\mu \mu \Gamma^\nu \nu \mathcal{F}^\lambda = \sigma^\mu \mu \overline{\Gamma \mathcal{F}}^\lambda$$

$$\boxed{\begin{array}{c} A \\ \Gamma \mathcal{F} \\ \Psi_C^* \end{array}}_A = \boxed{\begin{array}{c} A \\ \boxed{\begin{array}{c} B \\ \Gamma \mathcal{F} \\ \Psi_C^* \end{array}}_B \end{array}}_A$$

$$\text{LHS} = \begin{bmatrix} A & \Gamma \mathcal{F} & \Psi_C^* \\ \Psi_C^* & C \Gamma_{-A}^1 & \Gamma_{-A}^1 \end{bmatrix} = \begin{bmatrix} A & \Gamma_B^B & B \mathcal{F}_C & \Psi_C^* \\ \Psi_C^* & C \Gamma_{-B}^1 & B \mathcal{F}_C & \Gamma_{-A}^1 \end{bmatrix} = \boxed{\begin{array}{c} A \\ B \mathcal{F}_C & \Psi_C^* \\ \Psi_C^* & C \Gamma_{-B}^1 \end{array}}_A = \text{RHS}$$

$$\mu \begin{bmatrix} \Gamma : L | \mathcal{H} \\ \mathcal{Z}^* : \mathcal{H} \end{bmatrix} = \begin{bmatrix} \mu \mathcal{J}^{\nu A} \mathcal{J}_B^{\nu B} \mathcal{H} \\ \mu \mathcal{J}^{\nu} \mathcal{J}_B^* \mathcal{J}_{-A}^{B^{-1}} \end{bmatrix}$$

$$\left\{ \begin{array}{ll} \mathcal{J}_A^{\nu} \begin{bmatrix} \Gamma : L | \mathcal{H} \\ \mathcal{Z}^* \end{bmatrix} = 0 & \mathcal{J}_A^{\nu} \begin{bmatrix} \Gamma : L | \mathcal{H} \\ \mathcal{Z}^* \end{bmatrix} = 0 \\ \mathcal{J}_B^A \begin{bmatrix} \Gamma : L | \mathcal{H} \\ \mathcal{Z}^* \end{bmatrix} = {}^A \mathcal{J}_B & \mathcal{J}_A^B \begin{bmatrix} \Gamma : L | \mathcal{H} \\ \mathcal{Z}^* \end{bmatrix} = {}^{B^{-1}} \mathcal{J}_{-A} \end{array} \right.$$

$$\Rightarrow \text{LHS} = \mu \mathcal{J}^{\nu} \frac{\mathcal{J}_A^{\nu} \begin{bmatrix} \Gamma : L | \mathcal{H} \\ \mathcal{Z}^* \end{bmatrix} + \mathcal{J}_B^A \begin{bmatrix} \Gamma : L | \mathcal{H} \\ \mathcal{Z}^* \end{bmatrix} \mathcal{J}_B^B}{\mathcal{J}_A^{\nu} \begin{bmatrix} \Gamma : L | \mathcal{H} \\ \mathcal{Z}^* \end{bmatrix} + \mathcal{J}_B^* \mathcal{J}_A^B \begin{bmatrix} \Gamma : L | \mathcal{H} \\ \mathcal{Z}^* \end{bmatrix}} = \mu \mathcal{J}^{\nu} \begin{bmatrix} {}^A \mathcal{J}_B^{\nu} \mathcal{J}_B^B \mathcal{H} \\ \mathcal{J}_B^* \mathcal{J}_{-A}^{B^{-1}} \end{bmatrix} = \text{RHS}$$

$$\begin{array}{c}
x \\
\boxed{\begin{array}{c}
\boxed{\begin{array}{c}
A \\
\Gamma : \mathbb{L} \\
\mathbb{N}_C^*
\end{array}} : \boxed{\begin{array}{c}
B \\
\Gamma : \mathbb{L} \\
\mathbb{N}_C^* : \mathbb{N}_D^*
\end{array}} \\
A \quad \mu \quad B
\end{array}}
\end{array}
\quad \xrightarrow{\text{Poincaré invariance}} \quad
\begin{array}{c}
x | \Gamma : \mathbb{L} \\
\boxed{\begin{array}{c}
C \\
\mathbb{N}_C^* : \mathbb{N}_D^*
\end{array}}
\end{array}$$

$$\begin{aligned}
\Gamma_A^{-1} A \sigma_B^\mu \mu \Gamma^\nu D \Gamma_B^{-1} &= \Gamma_A^{-1} A \Gamma_E E \sigma_F^\nu B \Gamma_F^{-1} D \Gamma_B^{-1} = \underbrace{C \Gamma_E^{-1}}_E E \sigma_F^\nu \underbrace{D \Gamma_F^{-1}}_F = C \sigma_D^\nu \\
\Gamma_C^{-1} A \tilde{\sigma}_B^\mu \mu \Gamma^\nu B \Gamma_D^{-1} &= \Gamma_C^{-1} A \Gamma_E^{-1} E \tilde{\sigma}_F^\nu F \Gamma_B^{-1} B \Gamma_D^{-1} = \underbrace{E \Gamma_C^{-1}}_C E \tilde{\sigma}_F^\nu \underbrace{F \Gamma_D^{-1}}_D = C \tilde{\sigma}_D^\nu \\
\Rightarrow \text{LHS} &= \left[\begin{array}{c} A \Gamma_C \mathbb{N}_C^* \\ \mathbb{N}_C^* \Gamma_A^{-1} \end{array} \right] : \left[\begin{array}{c} \mu \Gamma^\nu B \Gamma_D \nu \mathbb{N}_D^* \\ \mu \Gamma^\nu \nu \mathbb{N}_D^* \Gamma_B^{-1} \end{array} \right] \\
&= \mathbb{N}_C \Gamma_C^{-1} \underbrace{A \sigma_B^\mu \Gamma^\nu \mathbb{N}_D^* D \Gamma_B^{-1} - m^A \Gamma_D \mathbb{N}_D^*}_C + \mathbb{N}_C^* \underbrace{A \tilde{\sigma}_B^\mu \Gamma^\nu B \Gamma_D^{-1} \nu \mathbb{N}_D^* - m^A \mathbb{N}_D^* \Gamma_A^{-1}}_C \\
&= \mathbb{N}_C \underbrace{C \sigma_D^\nu \mathbb{N}_D^* - m^C \mathbb{N}_C^*}_C + \mathbb{N}_C^* \underbrace{C \tilde{\sigma}_D^\nu \mathbb{N}_D^* - m^C \mathbb{N}_C^*}_C = \text{RHS}
\end{aligned}$$