$$\boxed{\mathfrak{H}|\mathfrak{H}:}^{\mu} = \underbrace{\mathfrak{H}^{\nu}\,\underline{\mathfrak{H}}^{i}_{\nu a} + \boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}}\ \overset{a}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} - \mathfrak{H}^{\mu}\ \boxed{\mathfrak{H}:} = \boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}\ \overset{a}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} + \mathfrak{H}^{\nu}\ \overbrace{\underline{\mathfrak{H}}^{i}_{\nu a}\ \overset{a}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} - {}_{\nu}\delta^{\mu}\boxed{\mathfrak{H}:}}$$

$$\boxed{\mathfrak{H}|\mathfrak{H}:}^{\mu} = \underbrace{\mathfrak{H}^{\nu}\,\underline{\mathfrak{H}}^{i}_{\nu a} + \boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}}\ \overset{a}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} - \mathfrak{H}^{\mu}\ \boxed{\mathfrak{H}:} = \boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}\ \overset{a}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} + \mathfrak{H}^{\nu}\ \overbrace{\underline{\mathfrak{H}}^{i}_{\nu a}\ \overset{a}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} - {}_{\nu}\delta^{\mu}\boxed{\mathfrak{H}:}}$$

$$\underset{\mu}{\boxed{\mathfrak{H}|\mathfrak{H}:}^{\mu}} \overset{\text{conserv}}{\underset{\text{elec current}}{=}} 0$$

$$\text{LHS} = \underset{\mu}{\overset{x}{\boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}}\ \overset{a}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} + \mathfrak{H}^{\nu}\ \overbrace{\underline{\mathfrak{H}}^{i}_{\nu a}\ \overset{a}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} - {}_{\nu}\delta^{\mu}\boxed{\mathfrak{H}:}}}$$

$$= \overset{x}{\boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}}\ \underset{\mu}{\overset{x}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu}}_{*} + \underset{\mu}{\overset{x}{\boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}}}_{**}\ \overset{x}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} + {}^{x}_{\mu}\mathfrak{H}^{\nu}\ \underbrace{\underline{\mathfrak{H}}^{i}_{\nu a}\ \overset{x}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} - {}_{\nu}\delta^{\mu}\boxed{\mathfrak{H}:}} + {}^{x}\mathfrak{H}^{\nu}\ \underset{\mu}{\overset{x}{\underset{i}{\underline{\mathfrak{H}}^{i}_{\nu a}\ \boxed{\mathfrak{H}:}}}^{\mu} - {}_{\nu}\delta^{\mu}\boxed{\mathfrak{H}:}}}_{***}$$

$$\underset{\text{harm}}{=} \overset{x}{\boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}}\ \overset{x}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu}_{*} + \underbrace{\underset{\mu}{\overset{x}{\boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}}} + {}^{x}_{\mu}\underline{\mathfrak{H}}^{j}_{b}\ \overset{x}{\underset{j}{\boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}}}}_{**}\ \overset{x}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} + {}^{x}_{\mu}\mathfrak{H}^{\nu}\ \underbrace{\underline{\mathfrak{H}}^{i}_{\nu a}\ \overset{x}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} - {}_{\nu}\delta^{\mu}\boxed{\mathfrak{H}:}} - {}^{x}\mathfrak{H}^{\nu}\ \underset{\nu}{\overset{x}{\underset{***}{\boxed{\mathfrak{H}:}}}}$$

$$= \overset{x}{\boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}}\ \overset{x}{\underset{i}{\boxed{\mathfrak{H}:}}} + \underbrace{{}^{x}_{\mu}\mathfrak{H}^{\nu}\,\underline{\mathfrak{H}}^{i}_{\nu a} + \underset{\mu}{\overset{x}{\boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}}} + {}^{x}_{\mu}\underline{\mathfrak{H}}^{j}_{b}\ \overset{x}{\underset{j}{\boxed{\mathfrak{H}|\mathfrak{H}}^{i}_{a}}}}\ \overset{x}{\underset{i}{\boxed{\mathfrak{H}:}}}^{\mu} - {}^{x}_{\mu}\mathfrak{H}^{\mu}\ \boxed{\mathfrak{H}:} - {}^{x}\mathfrak{H}^{\nu}\ \underset{\nu}{\overset{x}{\boxed{\mathfrak{H}:}}}$$

$$= \overset{x}{\boxed{\mathfrak{H}|{}^{x}\mathfrak{H}}^{i}_{a}}\ \overset{x}{\underset{i}{\boxed{{}^{x}\mathfrak{H}:{}^{x}_{-}\mathfrak{H}}}} + \underbrace{{}^{x}_{\mu}\mathfrak{H}^{\nu}\,\underline{\mathfrak{H}}^{i}_{\nu a} + \underset{\mu}{\overset{x}{\boxed{\mathfrak{H}|{}^{x}\mathfrak{H}}^{i}_{a}}} + {}^{x}_{\mu}\underline{\mathfrak{H}}^{j}_{b}\ \overset{x}{\underset{j}{\boxed{\mathfrak{H}|{}^{x}\mathfrak{H}}^{i}_{a}}}}\ \overset{x}{\underset{i}{\boxed{{}^{x}\mathfrak{H}:{}^{x}_{-}\mathfrak{H}}}}^{\mu} - {}^{x}_{\mu}\mathfrak{H}^{\mu}\ \boxed{{}^{x}\mathfrak{H}:{}^{x}_{-}\mathfrak{H}} - {}^{x}\mathfrak{H}^{\nu}\ \underset{\nu}{\overset{x}{\boxed{{}^{x}\mathfrak{H}:{}^{x}_{-}\mathfrak{H}}}} \overset{\text{Lie alg}}{\underset{\text{inv}}{=}} 0$$

$$\text{conserved el charge } \partial_t \int_S^{ds} \boxed{\mathfrak{H}|\mathfrak{H}:}^{0} = 0$$

$$0 = \partial_{\mu}\boxed{\mathfrak{H}|\mathfrak{H}:}^{\mu} = \mathfrak{d}\cdot\overset{-}{\boxed{\mathfrak{H}|\mathfrak{H}:}} + \partial_t\boxed{\mathfrak{H}|\mathfrak{H}:}^{0}$$

$$\Longrightarrow 0 = \int_S^{ds}\partial_{\mu}\boxed{\mathfrak{H}|\mathfrak{H}:}^{\mu} = \underbrace{\int_S^{ds}\mathfrak{d}\cdot\overset{-}{\boxed{\mathfrak{H}|\mathfrak{H}:}}}_{=0} + \int_S^{ds}\partial_t\boxed{\mathfrak{H}|\mathfrak{H}:}^{0} = \partial_t\int_S^{ds}\boxed{\mathfrak{H}|\mathfrak{H}:}^{0}$$