

$$x \left\{ \begin{array}{c} \overline{\mathcal{L} \times \mathcal{N}}_a^i \\ \overline{\mathcal{L} \times \mathcal{N}}_a^i \\ \mu \end{array} \right\} = x \left\{ \begin{array}{c} \overline{\mathcal{L} \times \mathcal{N}}_a^i \\ x \overline{\mathcal{L} \times \mathcal{N}}_a^i + x \overline{\mathcal{L} \times \mathcal{N}}_a^i \mathcal{N}_b^j \\ \mu \end{array} \right\} \stackrel{\text{group inv}}{=} \det x \overline{\mathcal{L}}_a^i x \overline{\mathcal{L}}_a^i \left\{ \begin{array}{c} \mathcal{N}_b^j \\ \mathcal{N}_b^j \end{array} \right\}$$

$$x \overline{\mathcal{L} \times \mathcal{N}}_a^i x \left\{ \begin{array}{c} a \\ \mathcal{N} \\ \mathcal{F} \\ i \end{array} \right\} + \overline{\mathcal{L} \times \mathcal{N}}_a^i + \overline{\mathcal{L} \times \mathcal{N}}_a^i \mathcal{N}_b^j + x \overline{\mathcal{L} \times \mathcal{N}}_a^i x \left\{ \begin{array}{c} \mu \\ \mathcal{N} \\ \mathcal{F} \\ i \end{array} \right\} \stackrel{\text{Lie alg inv}}{=} x \overline{\mathcal{L}}_a^i x \left\{ \begin{array}{c} \mathcal{N} \\ \mathcal{F} \end{array} \right\} + x \overline{\mathcal{L}}_a^i x \left\{ \begin{array}{c} \mathcal{N} \\ \mathcal{F} \\ \nu \end{array} \right\}$$

$$x \overline{\mathcal{L} \times \mathcal{N}}_a^i = \partial_t x \overline{\mathcal{L} \times \mathcal{N}}_a^i$$

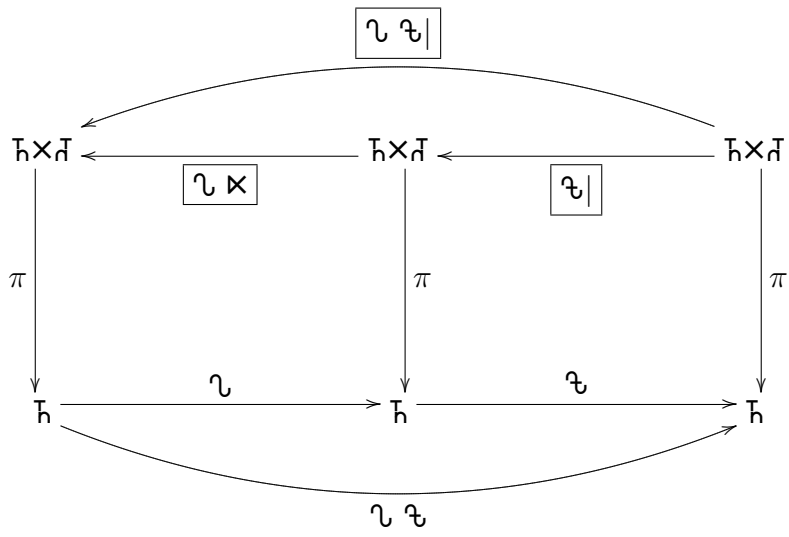
$$x \overline{\mathcal{L}} = \partial_t x \overline{\mathcal{L}} \Rightarrow \partial_t \overline{\mathcal{L}} = \underline{\partial_t \overline{\mathcal{L}}} = \underline{\partial_t \overline{\mathcal{L}}} = \underline{\overline{\mathcal{L}}} = x \overline{\mathcal{L}}$$

$$\text{LHS} = \partial_t x \left\{ \begin{array}{c} \overline{\mathcal{L} \times \mathcal{N}}_a^i \\ x \overline{\mathcal{L} \times \mathcal{N}}_a^i + x \overline{\mathcal{L} \times \mathcal{N}}_a^i \mathcal{N}_b^j \\ \mu \end{array} \right\} = \partial_t \overline{\mathcal{L} \times \mathcal{N}}_a^i x \overline{\mathcal{L}}_a^i \left\{ \begin{array}{c} \mathcal{N} \\ \mathcal{F} \end{array} \right\} = \partial_t \overline{\mathcal{L} \times \mathcal{N}}_a^i x \left\{ \begin{array}{c} \mathcal{N} \\ \mathcal{F} \end{array} \right\} + \partial_t x \overline{\mathcal{L}}_a^i \left\{ \begin{array}{c} \mathcal{N} \\ \mathcal{F} \end{array} \right\} = \text{RHS}$$

$$x^\nu : \mathcal{N}_a^i : \mu \mathcal{N}_a^i \in \mathbb{R}^d \times \mathbb{R}_M^N \times_d \mathbb{R}_M^N \xrightarrow[\text{el current}]{\boxed{\mathcal{L}|\mathcal{N}:\mathcal{F}}^\mu} \mathbb{R} \ni \boxed{\mathcal{L}|\mathcal{N}:\mathcal{F}}^\mu$$

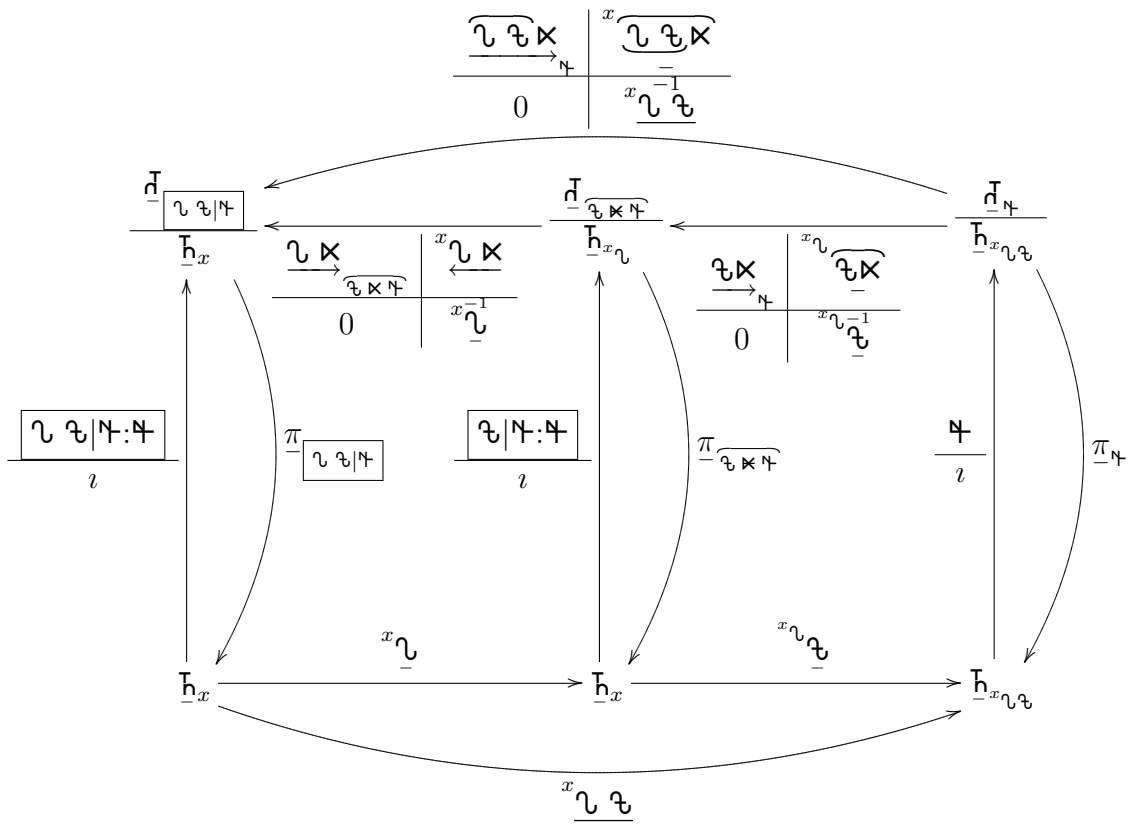
$$x \boxed{\mathcal{L}|\mathcal{N}:\mathcal{F}}^\mu = \overline{\mathcal{L} \times \mathcal{N}}_a^i + \overline{\mathcal{L} \times \mathcal{N}}_a^i \mathcal{N}_b^j x \left\{ \begin{array}{c} \mathcal{N} \\ \mathcal{F} \\ i \end{array} \right\} - x \overline{\mathcal{L}}_a^i x \left\{ \begin{array}{c} \mathcal{N} \\ \mathcal{F} \end{array} \right\} = \overline{\mathcal{L} \times \mathcal{N}}_a^i x \left\{ \begin{array}{c} \mathcal{N} \\ \mathcal{F} \\ i \end{array} \right\} + x \overline{\mathcal{L}}_a^i \overline{\mathcal{L} \times \mathcal{N}}_a^i \left\{ \begin{array}{c} \mathcal{N} \\ \mathcal{F} \\ i \end{array} \right\} - \delta^\mu x \left\{ \begin{array}{c} \mathcal{N} \\ \mathcal{F} \end{array} \right\}$$

$$\begin{array}{ccc} \mathfrak{h} \times \mathfrak{d} & \xleftarrow{\mathcal{L} \times} & \mathfrak{h} \times \mathfrak{d} \ni \mathcal{N} \\ \pi \downarrow & & \downarrow \pi \\ x \in \mathfrak{h} & \xrightarrow{\mathcal{L}} & \mathfrak{h} \ni x \overline{\mathcal{L}} \\ & & \overline{\mathcal{L} \times \mathcal{N}}_a^i \end{array}$$



$$\begin{array}{c}
 x \times d \xleftarrow{x \overbrace{l \ x}} x \times d \xleftarrow{x \overbrace{r \ |}} x \times d = x \overbrace{r \ |} \times d \\
 \xleftarrow{x \overbrace{l \ r \ |}}
 \end{array}$$

$$x \overbrace{l \ r \ |}^i_a = x \overbrace{l \ x \overbrace{r \ |}^j}_a^i = x \overbrace{l \ | \overbrace{r \ x \ |}^j}_b^i_a$$



$$x \begin{array}{|c|} \hline \overline{\mathcal{L} \mathcal{C} | \mathcal{T} : \mathcal{F}}^i \\ \hline \mu \\ \hline \end{array} = x \begin{array}{|c|} \hline \overline{\mathcal{L} \mid \overline{\mathcal{C} \mathcal{K} \mathcal{T}} : \overline{\mathcal{C} | \mathcal{T} : \mathcal{F}}^i} \\ \hline \mu \quad \nu \\ \hline \end{array}$$

$$\frac{\overline{\mathcal{L} \mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{k} a^i}{0} \mid \frac{x \overline{\mathcal{L} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{\lambda} a^i}{x \mathcal{L}^{-1} \mathcal{C}} = \frac{\overline{\mathcal{L} \mathcal{C} \mathcal{K}}}{0} \mid \frac{x \overline{\mathcal{L} \mathcal{C} \mathcal{K}}}{x \mathcal{L}^{-1} \mathcal{C}} = \frac{\overline{\mathcal{L} \mathcal{K}}}{0} \mid \frac{x \overline{\mathcal{L} \mathcal{K}}}{x \mathcal{L}^{-1}} \mid \frac{\overline{\mathcal{C} \mathcal{K}}}{0} \mid \frac{x \overline{\mathcal{C} \mathcal{K}}}{x \mathcal{L}^{-1} \mathcal{C}}$$

$$= \frac{\overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^b \xrightarrow{j} a^i}{0} \mid \frac{x \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{\lambda} a^i}{x \mathcal{L}^{-1}} \mid \frac{\overline{\mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{k} b^j}{0} \mid \frac{x \overline{\mathcal{C} \mathcal{K} \mathcal{T}}^j \xrightarrow{\lambda} b^j}{x \mathcal{L}^{-1} \mathcal{C}}$$

$$\Rightarrow \begin{cases} \frac{x \overline{\mathcal{L} \mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{k} a^i}{k} = \frac{x \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^b \xrightarrow{j} a^i}{j} \frac{x \overline{\mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{k} b^j}{k} \\ \frac{x \overline{\mathcal{L} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{\lambda} a^i}{\lambda} = \frac{x \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{j} a^i}{j} \frac{x \overline{\mathcal{C} \mathcal{K} \mathcal{T}}^j \xrightarrow{\lambda} b^j}{\lambda} + x \mathcal{L}^{-1} \mathcal{C} \frac{x \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{\lambda} a^i}{\lambda} \end{cases}$$

$$\Rightarrow x \mathcal{L}^{-1} \mu \text{ LHS} = x \mathcal{L}^{-1} \mu \frac{x \mathcal{L} \mathcal{C} \lambda \overline{\mathcal{L} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{\lambda} a^i + \overline{\mathcal{L} \mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{k} a^i \mathcal{F}^k}{\lambda}$$

$$= x \mathcal{L}^{-1} \mu \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^b \xrightarrow{j} a^i \frac{x \overline{\mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{k} b^j}{\lambda} + x \mathcal{L}^{-1} \mu \overline{\mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{k} b^j \frac{x \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{j} a^i}{j} + \frac{x \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{j} a^i}{j} \frac{x \overline{\mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{k} b^j}{\lambda} \mathcal{F}^k$$

$$= x \mathcal{L}^{-1} \mu \overline{\mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{\lambda} b^j \frac{x \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{j} a^i}{j} + \frac{x \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{j} a^i}{j} \frac{x \overline{\mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{k} b^j}{\lambda} \mathcal{F}^k + x \mathcal{L}^{-1} \mu \frac{x \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{j} a^i}{j} \left(= \nu \delta^z \right) \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{\lambda} b^j \mathcal{F}^k$$

$$= \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^b \xrightarrow{j} a^i + \frac{x \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{j} a^i}{j} x \mathcal{L}^{-1} \mu \overline{\mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{\lambda} b^j + \overline{\mathcal{C} \mathcal{K} \mathcal{T}}^c \xrightarrow{k} b^j \mathcal{F}^k$$

$$= \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^b \xrightarrow{j} a^i + \frac{x \overline{\mathcal{L} \mathcal{K} \mathcal{C} \mathcal{K} \mathcal{T}}^i \xrightarrow{j} a^i}{j} \overline{\mathcal{C} | \mathcal{T} : \mathcal{F}}^j \xrightarrow{\nu} b^j = x \mathcal{L}^{-1} \mu \text{ RHS}$$

$$x \begin{array}{|c|} \hline \overline{\mathcal{L} | \mathcal{T}}^i \\ \hline a \\ \hline \end{array} = x \begin{array}{|c|} \hline \overline{\mathcal{L} | x \mathcal{L} \mathcal{T}}^i \\ \hline a \\ \hline \end{array}$$

$$x \begin{array}{|c|} \hline \overline{\mathcal{L} : \mathcal{T}}^i \\ \hline a \\ \hline \end{array} = \frac{d}{dt} x \begin{array}{|c|} \hline \overline{\mathcal{L}_t : \mathcal{T}}^i \\ \hline a \\ \hline \end{array} = x \mathcal{F} \underbrace{\overline{\mathcal{L} : \mathcal{T}}^i \xrightarrow{\nu} a^i + x \mathcal{F} \overline{\mathcal{L} : \mathcal{T}}^i \xrightarrow{j} b^j}_{j} = x \mathcal{F} \overline{\mathcal{L} : \mathcal{T}}^i \xrightarrow{j} b^j$$