

$$\mathbb{H}_{\triangleleft \mathbb{K}} \rtimes \mathbb{H} \leq \mathbb{H}_{\triangleleft \mathbb{K}} \rtimes \mathbb{H}_{\triangleleft \mathbb{K}} \xrightarrow{\quad} \mathbb{H}_{\triangleleft \mathbb{K}} \xleftarrow{\quad} \mathbb{H} \leq \mathbb{H}_{\triangleleft \mathbb{K}} \rtimes \mathbb{H}_{\triangleleft \mathbb{K}}$$

$$\overbrace{k \rtimes \lambda}^y = \sum_x^{|y|} k^x \lambda^y = \sum_{x \leq y} k^x \lambda^y \text{ endl}$$

$$\underbrace{\lambda \rtimes \gamma}_x = \sum_y^{|\lambda|} \lambda^y \gamma = \sum_{x \leq y} \lambda^y \gamma \text{ endl}$$

$$\underbrace{k \rtimes \lambda}_x \rtimes \lambda = k \rtimes \underbrace{\lambda * \lambda}_x$$

$$\begin{aligned} \overbrace{\underbrace{k \rtimes \lambda}_x \rtimes \lambda}^z &= \sum_{y \leq z} \overbrace{k \rtimes \lambda}^y \lambda^z = \sum_{y \leq z} \sum_{x \leq y} k^x \lambda^y \lambda^z = \sum_{x \leq y \leq z} k^x \lambda^y \lambda^z \\ &\stackrel{\text{Fub}}{=} \sum_{x \leq z} k^x \sum_{x \leq y \leq z} \lambda^y \lambda^z = \sum_{x \leq z} k^x \underbrace{\lambda * \lambda}_x^z = \overbrace{k \rtimes \underbrace{\lambda * \lambda}_x}^z \end{aligned}$$

$$k \rtimes I = k$$

$$\overbrace{k \rtimes I}^y = \sum_{x \leq y} k^x \underbrace{x I^y}_{=0 \text{ } \curvearrowright \text{ } x \neq y} = k^y \underbrace{y I^y}_{=1} = k^y$$

$$\text{inv } \lambda \in \mathbb{H} \leq \mathbb{H}_{\triangleleft \mathbb{K}} \Rightarrow \underbrace{k \rtimes \lambda}_x \rtimes \lambda^{-1} = k$$

$$k \rtimes \lambda = k \Leftrightarrow k = k \rtimes \lambda^{-1}$$