$$\hbar \leqslant \hbar_{\triangle}\mathbb{K} = \begin{cases} \hbar\times\hbar \xrightarrow{\wr} \mathbb{K} \\ \mathrm{Trg}\ \wr \subset P \\ {}_x\wr^y \neq 0 \curvearrowright x{:}y \in P \end{cases}$$

$$\wr \ast \mathfrak{z} \in \hbar \leqslant \hbar_{\triangle}\mathbb{K} \xleftarrow[\text{bilin}]{\ast} \hbar \leqslant \hbar_{\triangle}\mathbb{K} \bowtie \hbar \leqslant \hbar_{\triangle}\mathbb{K} \ni \wr : \mathfrak{z}$$

$${}_x\underbrace{\wr \ast \mathfrak{z}}^z = \sum_{y}^{x|z} {}_x\wr^y \ {}_y\mathfrak{z}^z$$

$$x{:}z \notin P \Rightarrow x|z = \varnothing \underset{\text{void sum}}{\Rightarrow} {}_x\underbrace{\wr \ast \mathfrak{z}}^z = 0$$

$$\underbrace{\wr \ast \mathfrak{z}} \ast \mathfrak{t} \underset{\text{assoc}}{=} \wr \ast \underbrace{\mathfrak{z} \ast \mathfrak{t}}$$

$${}_x\overbrace{\underbrace{\wr \ast \mathfrak{z}} \ast \mathfrak{t}}^w = \sum_{z}^{x|w} {}_x\overbrace{\wr \ast \mathfrak{z}}^z \ {}_z\mathfrak{t}^w = \sum_{x \leqslant y \leqslant z \leqslant w} {}_x\wr^y \ {}_y\mathfrak{z}^z \ {}_z\mathfrak{t}^w = \sum_{z}^{x|w} \sum_{y}^{x|z} \underbrace{{}_x\wr^y \ {}_y\mathfrak{z}^z} \ {}_z\mathfrak{t}^w$$

$$= \sum_{y}^{x|w} \sum_{z}^{y|w} {}_x\wr^y \ \underbrace{{}_y\mathfrak{z}^z \ {}_z\mathfrak{t}^w} = \sum_{y}^{x|w} {}_x\wr^y \ {}_y\overbrace{\mathfrak{z} \ast \mathfrak{t}}^w = {}_x\overbrace{\wr \ast \underbrace{\mathfrak{z} \ast \mathfrak{t}}}^w$$

$$\hbar \leqslant \hbar_{\triangle}\mathbb{K} \ni \wr \ \text{inv} \ \Leftrightarrow \begin{cases} \bigvee \overset{-1}{\wr} \in \hbar \leqslant \hbar_{\triangle}\mathbb{K} \\ \wr \ast \overset{-1}{\wr} = I = \overset{-1}{\wr} \ast \wr \end{cases}$$

$$\operatorname{inv}\ \wr \in {}^{\hbar}\leqslant{}^{\hbar}\triangle\mathbb{K} \Leftrightarrow \bigwedge_{x}^{\hbar} {}_{x}\wr^{x} \neq 0$$

$$x<y \Rightarrow \begin{cases} {}_{x}\wr^{x}\ {}_{x}\overset{-1}{\wr}{}^{y} = -\sum\limits_{x\,<\,z\,\leqslant\,y} {}_{x}\wr^{z}\ {}_{z}\overset{-1}{\wr}{}^{y} \\ \sum\limits_{x\,\leqslant\,z\,\leqslant\,y} {}_{x}\wr^{z}\ {}_{z}\overset{-1}{\wr}{}^{y} = 0 \end{cases}$$

$$\Rightarrow:\quad 1 = {}_{x}I^{x} = {}_{x}\overbrace{\wr * \overset{-1}{\wr}}^{x} = \sum_{y}^{x|x} {}_{x}\wr^{y}\ {}_{y}\overset{-1}{\wr}{}^{x} = {}_{x}\wr^{x}\ {}_{x}\overset{-1}{\wr}{}^{x} \Rightarrow {}_{x}\wr^{x} \neq 0$$

$$\Leftarrow:\quad {}_{x}\overset{-1}{\wr}{}^{y}\quad \text{ind def } n = \sharp x|y \geqslant 1$$

$$\sharp x|y = 1 \Rightarrow x = y \Rightarrow {}_{x}\overset{-1}{\wr}{}^{x} = \frac{1}{{}_{x}\wr^{x}}$$

$$1 < \sharp x|y \Rightarrow \bigwedge_{x\,<\,z\,\leqslant\,y} \sharp z|y < \sharp x|y \underset{\text{ind}}{\Longrightarrow} {}_{z}\overset{-1}{\wr}{}^{y}\quad \text{well-def}$$

$${}_{x}\overset{-1}{\wr}{}^{y} \underset{*}{=} -\frac{1}{{}_{x}\wr^{x}} \sum_{x\,<\,z\,\leqslant\,y} {}_{x}\wr^{z}\ {}_{z}\overset{-1}{\wr}{}^{y}$$

$$\Rightarrow {}_{x}\overbrace{\wr * \overset{-1}{\wr}}^{y} = \sum_{x\,\leqslant\,z\,\leqslant\,y} {}_{x}\wr^{z}\ {}_{z}\overset{-1}{\wr}{}^{y} = {}_{x}\wr^{x}\ {}_{x}\overset{-1}{\wr}{}^{y} + \sum_{x\,<\,z\,\leqslant\,y} {}_{x}\wr^{z}\ {}_{z}\overset{-1}{\wr}{}^{y} \underset{*}{=} -\sum_{x\,<\,z\,\leqslant\,y} {}_{x}\wr^{z}\ {}_{z}\overset{-1}{\wr}{}^{y} + \sum_{x\,<\,z\,\leqslant\,y} {}_{x}\wr^{z}\ {}_{z}\overset{-1}{\wr}{}^{y} = 0 \underset{x\,<\,y}{=} {}_{x}I^{y}$$