$$\mathsf{C}|\Gamma$$

$$\Big\downarrow \asymp$$

$$\mathsf{C}|\Phi \triangledown \Gamma$$

$$\Phi \triangledown \Gamma \xrightarrow[\asymp]{\zeta} \acute{\Phi} \triangledown \acute{\Gamma} \Rightarrow \begin{cases} \alpha \in \Phi \xrightarrow[\asymp]{\varphi} \acute{\Phi} \ni \acute{\alpha} \\ \dim_{\Phi} \Gamma = \dim_{\acute{\Phi}} \acute{\Gamma} \\ \bigwedge\limits_{\dot{\Gamma} \in \Gamma}^{\text{basis}} \bigvee\limits_{\dot{\acute{\Gamma}} \in \acute{\Gamma}}^{\text{basis}} \underbrace{\alpha_i {}^{i}\Gamma}\, \Gamma := \acute{\alpha}_i {}^{i}\digamma \end{cases} \Rightarrow \begin{cases} \Gamma \xrightarrow[\asymp]{\Gamma} \acute{\Gamma} \\ \bigwedge\limits_{\Gamma \subset \Gamma} \Gamma^{\zeta} = \Gamma \Gamma \end{cases}$$

$$\dim_{\Phi} \Gamma = \dim_{\acute{\Phi}} \Gamma^{\zeta}$$

$$\dim_{\Phi} \Gamma = m \Rightarrow \bigvee \Gamma = \Gamma_m > \Gamma_{m-} > \cdots > \Gamma_1 > 0$$

$$\Rightarrow \overleftrightarrow{\Gamma} = \overleftrightarrow{\Gamma_m} > \overleftrightarrow{\Gamma_{m-}} > \cdots > \overleftrightarrow{\Gamma_1} > 0 \Rightarrow \dim_{\acute{\Phi}} \Gamma^{\zeta} \geqslant m = \dim_{\Phi} \Gamma^{\zeta \zeta^{-1}} \geqslant \dim_{\acute{\Phi}} \Gamma^{\zeta}$$

$$\overleftrightarrow{\Gamma} = 1^{\zeta} = \acute{1} = \acute{\Gamma}$$

$$ {}^{1}\Gamma \cdots {}^{d}\Gamma \underset{\text{basis}}{\in} \Gamma \Rightarrow \overleftrightarrow{\Phi\, {}^{i}\Gamma} = \acute{\Phi}\ {}^{i}\digamma \text{ 1-dim } \Rightarrow \overleftrightarrow{\Phi \frac{{}^{i}\Gamma}{i \in I}} = \overleftrightarrow{\bigvee_{i}^{I} \Phi\, {}^{i}\Gamma} = \bigvee_{i}^{I} \acute{\Phi}\ {}^{i}\digamma = \acute{\Phi} \frac{{}^{i}\digamma}{i \in I}$$

$$\acute{\Phi} \text{ frei } {}^{1}\acute{\Gamma} \cdots {}^{d}\acute{\Gamma}$$

$$\acute{\Gamma} = \overleftrightarrow{\Phi \left\{ {}^{1}\Gamma \cdots {}^{d}\Gamma \right\}} = \acute{\Phi} \left\{ {}^{1}\acute{\Gamma} \cdots {}^{d}\acute{\Gamma} \right\}$$

$$\bigwedge_{2\leqslant i\leqslant d}\bigvee \Phi \xrightarrow{\varphi_i} \Phi' \bigwedge_{\alpha}^{\Phi} \overline{1 \mid \alpha\ \Phi\frac{{}^1\mathfrak{f}}{{}^i\mathfrak{f}}} = \Phi' \underbrace{{}^1\mathfrak{f}' + \varphi_i(\alpha)\,{}^i\bar{\mathfrak{f}}'}$$

$$\alpha \neq 0 \Longrightarrow \Phi\,{}^1\mathfrak{f} : \Phi\,{}^i\mathfrak{f} \neq 1 \mid \alpha\ \Phi\frac{{}^1\mathfrak{f}}{{}^i\mathfrak{f}} \in \Phi\left\{{}^1\mathfrak{f} : {}^i\mathfrak{f}\right\} \Longrightarrow \Phi'\,{}^1\mathfrak{f}' : \Phi'\,{}^i\bar{\mathfrak{f}}' \neq \overline{1 \mid \alpha\ \Phi\frac{{}^1\mathfrak{f}}{{}^i\mathfrak{f}}} \in \Phi'\left\{{}^1\mathfrak{f}' : {}^i\bar{\mathfrak{f}}'\right\}$$

$$\Longrightarrow \bigvee_{\alpha_1 \neq 0 \neq \alpha_i}^{\text{eind}} \overline{1 \mid \alpha\ \Phi\frac{{}^1\mathfrak{f}}{{}^i\mathfrak{f}}} = \Phi' \underbrace{\alpha_1\,{}^1\mathfrak{f}' + \alpha_i\,{}^i\bar{\mathfrak{f}}'} = \Phi' \underbrace{{}^1\mathfrak{f}' + \varphi_i(\alpha)\,{}^i\bar{\mathfrak{f}}'}$$

$$\begin{cases} \varphi_i(0) = 0 \\ \varphi_i(1) = 1 \quad \underset{\text{OE}}{\Longleftarrow} \text{ replace } {}^i\bar{\mathfrak{f}}' \text{ by } \varphi_i(1)\,{}^i\bar{\mathfrak{f}}' \end{cases}$$

$$\varphi_i = \varphi_j$$

$$\lambda\underbrace{{}^1\mathfrak{f} + \alpha\,{}^i\mathfrak{f}} + \mu\underbrace{{}^1\mathfrak{f} + \alpha\,{}^i\mathfrak{f}} \in \Phi\left\{{}^i\mathfrak{f} : {}^j\mathfrak{f}\right\} \Leftrightarrow \lambda = -\mu \Longrightarrow \overbrace{1 \mid \alpha\ \Phi\frac{{}^1\mathfrak{f}}{{}^i\mathfrak{f}} \vee 1 \mid \alpha\ \Phi\frac{{}^1\mathfrak{f}}{{}^i\mathfrak{f}}} \wedge \Phi\left\{{}^i\mathfrak{f} : {}^j\mathfrak{f}\right\} = \Phi\left({}^i\mathfrak{f} - {}^j\mathfrak{f}\right)$$

$$\Longrightarrow \varphi_i(\alpha)\,{}^i\bar{\mathfrak{f}}' - \varphi_j(\alpha)\,{}^j\mathfrak{f}' = \underbrace{{}^1\mathfrak{f} + \varphi_i(\alpha)\,{}^i\bar{\mathfrak{f}}'} - \underbrace{{}^1\mathfrak{f} + \varphi_j(\alpha)\,{}^j\mathfrak{f}'}$$

$$\in \overbrace{\Phi'\underbrace{{}^1\mathfrak{f}' + \varphi_i(\alpha)\,{}^i\bar{\mathfrak{f}}'} \vee \Phi'\underbrace{{}^1\mathfrak{f}' + \varphi_j(\alpha)\,{}^j\bar{\mathfrak{f}}'}} \wedge \Phi'\left\{{}^i\bar{\mathfrak{f}}' : {}^j\mathfrak{f}'\right\} = \underbrace{\overline{1 \mid \alpha\ \Phi\frac{{}^1\mathfrak{f}}{{}^i\mathfrak{f}}} \vee \overline{\underbrace{{}^1\mathfrak{f} + \alpha\,{}^i\mathfrak{f}}}} \wedge \overline{\Phi\left\{{}^i\mathfrak{f} : {}^j\mathfrak{f}\right\}}$$

$$= \overline{\underbrace{\Phi\overbrace{{}^1\mathfrak{f} + \alpha\,{}^i\mathfrak{f}} \vee \Phi\overbrace{{}^1\mathfrak{f} + \alpha\,{}^i\mathfrak{f}}} \wedge \Phi\left\{{}^i\mathfrak{f} : {}^j\mathfrak{f}\right\}} = \overline{\Phi\left({}^i\mathfrak{f} - {}^j\mathfrak{f}\right)} \text{ 1-dim}$$

$$\Longrightarrow \varphi_i(\alpha)\,{}^i\bar{\mathfrak{f}}' - \varphi_j(\alpha)\,{}^j\mathfrak{f}' = \lambda\underbrace{\varphi_i(1)\,{}^i\bar{\mathfrak{f}}' - \varphi_j(1)\,{}^j\mathfrak{f}'} = \lambda\underbrace{{}^i\bar{\mathfrak{f}}' - {}^j\mathfrak{f}'} \Longrightarrow \varphi_i(\alpha) = \lambda = \varphi_j(\alpha)$$

$$\alpha \in \Phi \xrightarrow[\varphi_i = \varphi_j]{\cdot\varphi} \Phi' \ni \alpha' \Longrightarrow \begin{cases} 1' = 1 \\ \overbrace{1 \mid \alpha\ \Phi\frac{{}^1\mathfrak{f}}{{}^i\mathfrak{f}}}^{\zeta} = \Phi'\underbrace{{}^1\mathfrak{f}' + \alpha'\,{}^i\bar{\mathfrak{f}}'} \end{cases}$$

$$\bigwedge_{2\leqslant m\leqslant d} \overline{\Phi\,\underbrace{{}^{1}\text{ʃ} + \alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_m\,{}^{m}\text{ʃ}}} = \acute{\Phi}\,\underbrace{{}^{1}\text{ʃ}' + \acute{\alpha}_2\,{}^{2}\text{ʃ}'\,\ddot{+}\,\acute{\alpha}_m\,{}^{m}\text{ʃ}'}$$

$$2 = m: \quad \text{Def}: \quad 2\leqslant m-1 \curvearrowright m+1\leqslant d:$$

$$\text{LHS} = \overbrace{\Phi\,\underbrace{{}^{1}\text{ʃ} + \alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_{m-}\,{}^{m-}\text{ʃ}} \vee \Phi\,{}^{m}\text{ʃ}} \wedge \overbrace{\Phi\,\underbrace{{}^{1}\text{ʃ} + \alpha_m\,{}^{m}\text{ʃ}} \vee \Phi\left\{{}^{2}\text{ʃ}\cdots{}^{m-}\text{ʃ}\right\}} \Longrightarrow$$

$$\overline{\Phi\,\underbrace{{}^{1}\text{ʃ} + \alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_m\,{}^{m}\text{ʃ}}} = \overline{\Phi\,\underbrace{{}^{1}\text{ʃ} + \alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_{m-}\,{}^{m-}\text{ʃ}} \vee \Phi\,{}^{m}\text{ʃ}} \wedge \overline{\Phi\,\underbrace{{}^{1}\text{ʃ} + \alpha_m\,{}^{m}\text{ʃ}} \vee \Phi\left\{{}^{2}\text{ʃ}\cdots{}^{m-}\text{ʃ}\right\}}$$

$$= \underbrace{\overline{\Phi\,\underbrace{{}^{1}\text{ʃ} + \alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_{m-}\,{}^{m-}\text{ʃ}}} \vee \overline{\Phi\,{}^{m}\text{ʃ}}} \wedge \underbrace{\overline{\Phi\,\underbrace{{}^{1}\text{ʃ} + \alpha_m\,{}^{m}\text{ʃ}}} \vee \overline{\Phi\left\{{}^{2}\text{ʃ}\cdots{}^{m-}\text{ʃ}\right\}}} \underset{\text{Ind}}{=}$$

$$\underbrace{\overbrace{\acute{\Phi}\,\underbrace{{}^{1}\text{ʃ}' + \acute{\alpha}_2\,{}^{2}\text{ʃ}'\,\ddot{+}\,\acute{\alpha}_{m-}\,{}^{m-}\text{ʃ}'}} \vee \overbrace{\acute{\Phi}\,{}^{m}\text{ʃ}'}} \wedge \underbrace{\overbrace{\acute{\Phi}\,\underbrace{{}^{1}\text{ʃ}' + \acute{\alpha}_m\,{}^{m}\text{ʃ}'}} \vee \acute{\Phi}\left\{{}^{2}\text{ʃ}'\cdots{}^{m-}\text{ʃ}'\right\}} = \text{RHS}$$

$$\overline{\Phi\,\underbrace{\alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_d\,{}^{d}\text{ʃ}}} = \acute{\Phi}\,\underbrace{\acute{\alpha}_2\,{}^{2}\text{ʃ}'\,\ddot{+}\,\acute{\alpha}_d\,{}^{d}\text{ʃ}'}$$

$$\Phi\,\underbrace{\alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_d\,{}^{d}\text{ʃ}} = \overbrace{\Phi\,\underbrace{{}^{1}\text{ʃ} + \alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_d\,{}^{d}\text{ʃ}} \vee \Phi\,{}^{1}\text{ʃ}} \wedge \Phi\left\{{}^{2}\text{ʃ}\cdots{}^{d}\text{ʃ}\right\}$$

$$\Longrightarrow \overline{\Phi\,\underbrace{\alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_d\,{}^{d}\text{ʃ}}} = \overline{\underbrace{\Phi\,\overbrace{{}^{1}\text{ʃ} + \alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_d\,{}^{d}\text{ʃ}} \vee \Phi\,{}^{1}\text{ʃ}} \wedge \Phi\left\{{}^{2}\text{ʃ}\cdots{}^{d}\text{ʃ}\right\}}$$

$$= \overline{\underbrace{\alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_d\,{}^{d}\text{ʃ}}} = \overline{\underbrace{\Phi\,\overbrace{{}^{1}\text{ʃ} + \alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_d\,{}^{d}\text{ʃ}} \vee \Phi\,{}^{1}\text{ʃ}} \wedge \Phi\left\{{}^{2}\text{ʃ}\cdots{}^{d}\text{ʃ}\right\}}$$

$$\underbrace{\overline{\Phi\,\underbrace{{}^{1}\text{ʃ} + \alpha_2\,{}^{2}\text{ʃ}\,\ddot{+}\,\alpha_d\,{}^{d}\text{ʃ}}} \vee \overline{\Phi\,{}^{1}\text{ʃ}}} \wedge \overline{\Phi\left\{{}^{2}\text{ʃ}\cdots{}^{d}\text{ʃ}\right\}}$$

$$\underset{5/}{=} \overbrace{\acute{\Phi}\,\underbrace{{}^{1}\text{ʃ}' + \acute{\alpha}_2\,{}^{2}\text{ʃ}'\,\ddot{+}\,\acute{\alpha}_d\,{}^{d}\text{ʃ}'} \vee \acute{\Phi}\,{}^{1}\text{ʃ}'} \wedge \acute{\Phi}\left\{{}^{2}\text{ʃ}'\cdots{}^{d}\text{ʃ}'\right\} = \acute{\Phi}\,\underbrace{\acute{\alpha}_2\,{}^{2}\text{ʃ}'\,\ddot{+}\,\acute{\alpha}_d\,{}^{d}\text{ʃ}'}$$

$$\begin{cases} \overbrace{\alpha+\beta}^{\,\prime} & = \alpha' + \beta' \\ \overbrace{\alpha\beta}^{\,\prime} & = \alpha'\beta' \end{cases}$$

$$3 \leqslant d: \quad \Phi\underbrace{{}^1\daleth + \overbrace{\alpha+\beta}\,{}^2\daleth + {}^3\daleth} \curlyvee \Phi\underbrace{{}^1\daleth + \alpha\,{}^2\daleth} \vee \Phi\underbrace{{}^1\daleth + \beta\,{}^3\daleth}$$

$$\underset{5/6/1\prime=1}{\Longrightarrow} \quad \acute{\Phi}\,\underbrace{{}^1\acute{\daleth} + \overbrace{\alpha+\beta}^{\prime}\,{}^2\acute{\daleth} + {}^3\acute{\daleth}} \curlyvee \acute{\Phi}\,\underbrace{{}^1\acute{\daleth} + \acute{\alpha}\,{}^2\acute{\daleth}} \vee \acute{\Phi}\,\underbrace{{}^1\acute{\daleth} + \acute{\beta}\,{}^3\acute{\daleth}}$$

$$\Longrightarrow\ {}^1\acute{\daleth} + \overbrace{\alpha+\beta}^{\prime}\ {}^2\acute{\daleth} + {}^3\acute{\daleth} = \lambda\underbrace{{}^1\acute{\daleth} + \acute{\alpha}\,{}^2\acute{\daleth}} + \mu\underbrace{{}^1\acute{\daleth} + \acute{\beta}\,{}^3\acute{\daleth}} \Longrightarrow \lambda = \acute{1} = \mu \Longrightarrow \overbrace{\alpha+\beta}^{\prime} = \alpha' + \beta'$$

$$\Phi\underbrace{{}^1\daleth + \overbrace{\alpha\beta}\,{}^2\daleth + \alpha\,{}^3\daleth} \curlyvee \Phi\,{}^1\daleth \vee \Phi\underbrace{\beta\,{}^2\daleth + {}^3\daleth} \underset{5/6/1\prime=1}{\Longrightarrow} \acute{\Phi}\underbrace{{}^1\acute{\daleth} + \overbrace{\alpha\beta}^{\prime}\,{}^2\acute{\daleth} + \acute{\alpha}\,{}^3\acute{\daleth}} \curlyvee \acute{\Phi}\,{}^1\acute{\daleth} \vee \acute{\Phi}\underbrace{\acute{\beta}\,{}^2\acute{\daleth} + {}^3\acute{\daleth}}$$

$$\Longrightarrow\ {}^1\acute{\daleth} + \overbrace{\alpha\beta}^{\prime}\ {}^2\acute{\daleth} + \alpha'\ {}^3\acute{\daleth} = \lambda\,{}^1\acute{\daleth} + \mu\underbrace{\beta'\,{}^2\acute{\daleth} + {}^3\acute{\daleth}} \Longrightarrow \lambda = \acute{1}$$

$$\mu = \alpha' \Longrightarrow \overbrace{\alpha\beta}^{\,\prime} = \alpha'\,\beta'$$

$$\Phi \underset{\asymp}{\xrightarrow{\varphi}} \acute{\Phi}: \overbrace{\Phi\underbrace{{}^1\daleth + \alpha\,{}^2\daleth}}^{\zeta} = \acute{\Phi}\,\underbrace{{}^1\acute{\daleth} + \varphi(\alpha)\ {}^2\acute{\daleth}} \Longleftarrow \Phi \xrightarrow[\text{hom}]{\varphi} \acute{\Phi}$$

$$\overbrace{\acute{\Phi}\underbrace{{}^1\acute{\daleth} + \acute{\alpha}\,{}^2\acute{\daleth}}}^{\zeta^{-1}} = \Phi\underbrace{{}^1\daleth + \psi(\acute{\alpha})\ {}^2\daleth} \Longleftarrow \acute{\Phi} \xrightarrow[\text{hom}]{\psi} \Phi$$

$$\Longrightarrow \Phi\underbrace{{}^1\daleth + \alpha\,{}^2\daleth} = \overbrace{\overbrace{\Phi\underbrace{{}^1\daleth + \alpha\,{}^2\daleth}}^{\zeta}}^{\zeta^{-1}} = \overbrace{\acute{\Phi}\underbrace{{}^1\acute{\daleth} + \alpha^{\varphi}\,{}^2\acute{\daleth}}}^{\zeta^{-1}} = \Phi\underbrace{{}^1\daleth + \overbrace{\alpha^{\varphi}}^{\psi}\,{}^2\daleth} \Longrightarrow \alpha = \overbrace{\alpha^{\varphi}}^{\psi}$$

$$\text{analog } \acute{\alpha} = \overbrace{\acute{\alpha}^{\psi}}^{\varphi}$$

$$\underbrace{\alpha_i\,{}^i\daleth}\,\daleth := \acute{\alpha}_i\,{}^i\acute{\daleth} \Longrightarrow \underbrace{\alpha\daleth}\,\daleth = \alpha'\,\underbrace{\daleth\daleth}$$

$$\overbrace{\alpha\sum_i \alpha_i\,{}^i\daleth}\,\daleth = \overbrace{\sum_i \underbrace{\alpha\alpha_i}\,{}^i\daleth}\,\daleth = \sum_i \underbrace{\alpha\alpha_i}'\ {}^i\acute{\daleth} = \sum_i \alpha'\,\alpha_i'\ {}^i\acute{\daleth} = \alpha' \sum_i \alpha_i'\ {}^i\acute{\daleth} = \alpha'\ \underbrace{\overbrace{\sum_i \alpha_i\,{}^i\daleth}\,\daleth}$$

$$\text{ʃ} \sqsupset \text{ʃ} \Rightarrow {}^{\zeta}\text{ʃ} = \text{ʃ}\text{ʃ}$$

$$\text{OE} \ \dim_{\Phi} \text{ʃ} = 1 \Rightarrow \text{ʃ} = \Phi \overbrace{\sum_i \alpha_i {}^{i}\text{ʃ}}$$

$$\text{If } \alpha_1 \neq 0 \underset{\text{OE}}{\Rightarrow} \alpha_1 = 1 \Rightarrow {}^{\zeta}\text{ʃ} = \overbrace{\Phi \underbrace{{}^{1}\text{ʃ} + \alpha_2 {}^{2}\text{ʃ} \ddot{+} \alpha_d {}^{d}\text{ʃ}}}^{\zeta} \underset{5/}{=} \acute{\Phi} \underbrace{{}^{1}\acute{\text{ʃ}} + \acute{\alpha}_2 {}^{2}\acute{\text{ʃ}} \ddot{+} \acute{\alpha}_d {}^{d}\acute{\text{ʃ}}} = \text{ʃ}\text{ʃ}$$

$$\text{If } \alpha_1 = 0 \underset{6/}{\Rightarrow} {}^{\zeta}\text{ʃ} = \overbrace{\Phi \underbrace{\alpha_2 {}^{2}\text{ʃ} \ddot{+} \alpha_d {}^{d}\text{ʃ}}}^{\zeta} = \acute{\Phi} \underbrace{\acute{\alpha}_2 {}^{2}\acute{\text{ʃ}} \ddot{+} \acute{\alpha}_d {}^{d}\acute{\text{ʃ}}} = \text{ʃ}\text{ʃ}$$