$$0 \leqslant m \leqslant n: \quad \big|_m\, q^n = \frac{L \sqsubset q^n}{\dim_q L = m}$$

$$\big|\, q^n = \bigcup_m^{0|n} \big|_m\, q^n$$

$${}^{\complement}_{m}q^m \;\neg\; {}^{q^m}\!\blacktriangle q^n \underset{\asymp}{\longrightarrow} \big|_m\, q^n$$

$$\dot{\nu} \in {}^{\complement}_{m}q^m : \dot{L} \in {}^{q^m}\!\blacktriangle q^n \;\underset{\text{Ran}}{\Longrightarrow}\; q^m\ \dot{\nu}\ \ \dot{L} = q^m\ \dot{L} \sqsubset q^n$$

$${}^{\complement}_{m}q^m\ \dot{L} \mapsto q^m\ \dot{L} \sqsubset q^n$$

$$\sharp \big|_m\, q^n = \begin{bmatrix} n \\ m \end{bmatrix}_q = \prod_i^m \frac{q^n - q^i}{q^m - q^i} \text{ q-binomial}$$

$$\sharp \big|_m\, q^n = \frac{\sharp\, {}^{q^m}\!\blacktriangle q^n}{\sharp\, {}^{\complement}_{m}q^m} = \frac{\prod_i^m q^n - q^i}{\prod_i^m q^m - q^i} = \prod_i^m \frac{q^n - q^i}{q^m - q^i} = \begin{bmatrix} n \\ m \end{bmatrix}_q$$