

$$z \overline{z}^{-\nu/2} \int_{\mathbb{R}^d} \zeta \overline{\zeta} |z| \overline{\zeta} I = \nu(1-\alpha)/2 \int_{\mathbb{R}^d} \zeta \overline{\zeta} |z| \overline{\zeta} \left[\begin{array}{c|c|c} \frac{1-\alpha}{-1} & -1 & \alpha \\ \hline \alpha & 1 & -(1+\alpha) \end{array} \right] \overline{\zeta} |z|$$

$$M = A + 2B + D + (1 - \gamma) / 2$$

$$z \overline{z}^{-\nu/2} \int_{\mathbb{R}^d} \zeta \overline{\zeta} |z| \overline{\zeta} \int_{\mathbb{R}^d} \xi \overline{\xi} |z| \overline{\xi} \left[\begin{array}{c|c|c} -A & -B & A+B \\ \hline -B & -D & B+D \\ \hline A+B & B+D & -A-2B-D \end{array} \right] \overline{\zeta} |z| \overline{\xi} \\ = \left(\frac{\pi}{\nu M} \right)^{d/2} \int_{\mathbb{R}^d} \zeta \overline{\zeta} |z| \overline{\zeta} \left[\begin{array}{c|c|c} B^2 - AD - A(1-\gamma)/2 & AD - B^2 - B(1-\gamma)/2 & (A+B)(1-\gamma)/2 \\ \hline DA - B^2 - B(1-\gamma)/2 & B^2 - DA - D(1-\gamma)/2 & (B+D)(1-\gamma)/2 \\ \hline (A+B)(1-\gamma)/2 & (B+D)(1-\gamma)/2 & (A+2B+D)(\gamma-1)/2 \end{array} \right] \overline{\zeta} |z| \overline{\xi}$$

$$z \overline{z}^{-\nu/2} \int_{\mathbb{R}^d} \xi \overline{\xi} |z| \overline{\xi} = \exp \nu \left(\frac{\gamma}{2} z \overline{z} + (1-\gamma) z \xi + \frac{\gamma-1}{2} \xi \overline{\xi} \right) = \nu/2 \int_{\mathbb{R}^d} \xi \overline{\xi} |z| \overline{\xi} \left[\begin{array}{c|c|c} \frac{\gamma}{1-\gamma} & \frac{1-\gamma}{\gamma-1} & \end{array} \right] \overline{\xi}$$

$$\Rightarrow \text{LHS} = z \overline{z}^{-\nu/2} \int_{\mathbb{R}^d} \xi \overline{\xi} |z| \overline{\xi} \int_{\mathbb{R}^d} \zeta \overline{\zeta} |z| \overline{\zeta} \left[\begin{array}{c|c|c} -A & -B & A+B \\ \hline -B & -D & B+D \\ \hline A+B & B+D & -A-2B-D \end{array} \right] \overline{\zeta} |z| \overline{\xi}$$

$$= \nu/2 \int_{\mathbb{R}^d} \xi \overline{\xi} |z| \overline{\xi} \left[\begin{array}{c|c|c} A & B & \\ \hline B & D & \\ \hline -\nu & & \end{array} \right] \overline{\xi} \int_{\mathbb{R}^d} \zeta \overline{\zeta} |z| \overline{\zeta} \left[\begin{array}{c|c|c} A+B & & \\ \hline B+D & & \\ \hline (1-\gamma)/2 & & \end{array} \right] \overline{\zeta}$$

$$\stackrel{\text{real Gauss}}{=} \int_{\mathbb{R}^d} \xi \overline{\xi} |z| \overline{\xi} \left[\begin{array}{c|c|c} A & B & \\ \hline B & D & \\ \hline -\nu & & \end{array} \right] \overline{\xi} \left(\frac{\pi}{\nu M} \right)^{d/2} \int_{\mathbb{R}^d} \zeta \overline{\zeta} |z| \overline{\zeta} \left[\begin{array}{c|c|c} A+B & & \\ \hline B+D & & \\ \hline (1-\gamma)/2 & & \end{array} \right] \overline{\zeta} A+B | B+D | (1-\gamma)/2$$

$$= \left(\frac{\pi}{\nu M} \right)^{d/2} \int_{\mathbb{R}^d} \xi \overline{\xi} |z| \overline{\xi} \left[\begin{array}{c|c|c} (A+B)^2 - AM & (A+B)(B+D) - BM & (A+B)(1-\gamma)/2 \\ \hline (B+D)(A+B) - BM & (B+D)^2 - DM & (B+D)(1-\gamma)/2 \\ \hline (A+B)(1-\gamma)/2 & (B+D)(1-\gamma)/2 & (1-\gamma)^2/4 + M(\gamma-1)/2 \end{array} \right] \overline{\xi} = \text{RHS}$$

$$\frac{\alpha}{\zeta} I = \left(\frac{\nu(1-\gamma)^2}{2\pi(\alpha^2-\gamma)} \right)^{d/2} \int_{d\xi}^{\mathbb{R}_d} \frac{\gamma}{\xi} \zeta|\bar{\zeta}|\xi \frac{\begin{array}{c|c|c} -A & -B & A+B \\ -B & -D & B+D \\ \hline A+B & B+D & -A-2B-D \end{array}}{\zeta|\zeta|\xi}$$

$$2(\alpha^2-\gamma)A = (1-\alpha)^2\gamma: \quad 2(\alpha^2-\gamma)B = (1-\alpha)(\alpha-\gamma): \quad 2(\alpha^2-\gamma)D = (1-\alpha)^2$$

$$2(\alpha^2-\gamma)(A+2B+D) = (1-\alpha)(1+\alpha)(1-\gamma): \quad 4(\alpha^2-\gamma)(AD-B^2) = (1-\alpha)^2(\gamma-1)$$

$$2(\alpha^2-\gamma)M = 2(\alpha^2-\gamma)\left(A+2B+D+\frac{1-\gamma}{2}\right) = (1-\gamma)^2$$

$$4(\alpha^2-\gamma) \frac{B^2-AD-A\frac{1-\gamma}{2} \mid AD-B^2-B\frac{1-\gamma}{2}}{DA-B^2-B\frac{1-\gamma}{2} \mid B^2-DA-D\frac{1-\gamma}{2}} = \underbrace{(1-\alpha)(1-\gamma)^2}_{=2(\alpha^2-\gamma)M(1-\alpha)} \frac{1-\alpha \mid -1}{-1 \mid 0}$$

$$\Rightarrow \frac{1-\alpha}{2} \frac{1-\alpha \mid -1}{-1 \mid 0} \frac{\alpha}{1 \mid -(1+\alpha)} = \frac{1}{M} \frac{B^2-AD-A\frac{1-\gamma}{2} \mid AD-B^2-B\frac{1-\gamma}{2}}{DA-B^2-B\frac{1-\gamma}{2} \mid B^2-DA-D\frac{1-\gamma}{2}} \frac{(A+B)\frac{1-\gamma}{2}}{(B+D)\frac{1-\gamma}{2}} \frac{(A+2B+D)(\gamma-1)/2}{(A+B)\frac{1-\gamma}{2} \mid (B+D)\frac{1-\gamma}{2}}$$

$$\left(\frac{\nu M}{\pi} \right)^{d/2} = \left(\frac{\nu(1-\gamma)^2}{2\pi(\alpha^2-\gamma)} \right)^{d/2} \Rightarrow {}^z\text{LHS} = {}^z\text{RHS}$$

old

$$\begin{aligned}
\int_{d\zeta}^{\mathbb{R}^d} \alpha w + \bar{w} - (\alpha+1) \zeta/2 \left[\mathbf{e} \right]_{\zeta}^{\nu(1-\alpha)(1-\gamma)/(\alpha^2-\gamma)} z \left[\mathbf{e} \right]_{\zeta}^{-\gamma} &= z \left[\mathbf{e} \right]_{\bar{z}}^{\nu\gamma/2} \int_{d\zeta}^{\mathbb{R}^d} \zeta \left[\mathbf{e} \right]_{\zeta}^{-\nu(1-\gamma)/2} z \left[\mathbf{e} \right]_{\zeta}^{\nu(1-\gamma)} \alpha w + \bar{w} - (\alpha+1) \zeta/2 \left[\mathbf{e} \right]_{\zeta}^{\nu(1-\alpha)(1-\gamma)/(\alpha^2-\gamma)} \\
&= z \left[\mathbf{e} \right]_{\bar{z}}^{\nu\gamma/2} \int_{d\zeta}^{\mathbb{R}^d} \zeta \left[\mathbf{e} \right]_{\zeta}^{-\nu(1-\gamma)^2/2(\alpha^2-\gamma)} \alpha w + \bar{w} + (\alpha^2-\gamma) z/(1-\alpha) \left[\mathbf{e} \right]_{\zeta}^{\nu(1-\alpha)(1-\gamma)/(\alpha^2-\gamma)} \\
&= z \left[\mathbf{e} \right]_{\bar{z}}^{\nu\gamma/2} \alpha w + \bar{w} + (\alpha^2-\gamma) z/(1-\alpha) \left[\mathbf{e} \right]_{\bar{\alpha}\bar{w} + w + (\bar{\alpha}^2-\bar{\gamma}) \bar{z}/(1-\bar{\alpha})}^{\nu(1-\alpha)^2/2(\alpha^2-\gamma)} \\
&= \overbrace{w \left[\mathbf{e} \right]_{\bar{w}}^{(1-\alpha)\gamma/2} w \left[\mathbf{e} \right]_w^{\alpha-\gamma\bar{w}} \left[\mathbf{e} \right]_w^{1-\alpha/2}}^{-\nu(1-\alpha)/(\alpha^2-\gamma)} w \left[\mathbf{e} \right]_w^{-\nu(1-\alpha)} z \left[\mathbf{e} \right]_w^{\nu(1-\alpha)} \alpha z + (1-\alpha) w \left[\mathbf{e} \right]_{\bar{\alpha}\bar{z} + (1-\bar{\alpha})\bar{w}}^{\nu/2} = \overbrace{w \left[\mathbf{e} \right]_{\bar{w}}^{(1-\alpha)\gamma/2} w \left[\mathbf{e} \right]_w^{\alpha-\gamma\bar{w}} \left[\mathbf{e} \right]_w^{1-\alpha/2}}^{-\nu(1-\alpha)/(\alpha^2-\gamma)} z \overbrace{w}^{\alpha} I
\end{aligned}$$

$$\Re \frac{\gamma^2}{\alpha^2 - 2\alpha + \gamma} > 0 \implies$$

$$z \overbrace{\left[\frac{1-\alpha}{w} \right]}^{\text{Moyal restriction}} I \frac{\gamma^2}{\alpha^2 - 2\alpha + \gamma} \overbrace{\left[w \mathbf{e}_{\bar{w}}^{\alpha(1-\gamma)/2} w \mathbf{e}_w^{\gamma-\alpha} \bar{w} \mathbf{e}_w^{(1+\alpha)/2} \right]}^{d/2} \int_{\mathbb{R}^d} (1-\alpha)w + \bar{w} - (1-\alpha/2)\zeta \left[\mathbf{e}_{\zeta}^{\nu\alpha\gamma/(\alpha^2-2\alpha+\gamma)} \right] d\zeta / (2\pi)^{d/2} z \overbrace{\left[\frac{1-\gamma}{\zeta} \right]}^{\nu(\alpha^2-2\alpha+\gamma)}$$

$$\frac{\gamma^2}{\alpha^2 - 2\alpha + \gamma} \int_{\mathbb{R}^d} (1-\alpha)w + \bar{w} - (1-\alpha/2)\zeta \left[\mathbf{e}_{\zeta}^{\nu\alpha\gamma} \right] z \overbrace{\left[\frac{1-\gamma}{\zeta} \right]}^{\nu(\alpha^2-2\alpha+\gamma)} d\zeta / (2\pi)^{d/2}$$

$$= z \left[\mathbf{e}_{\bar{z}}^{\nu(\alpha^2-2\alpha+\gamma)(1-\gamma)/2} \right] \frac{\gamma^2}{\alpha^2 - 2\alpha + \gamma} \int_{\mathbb{R}^d} \zeta \left[\mathbf{e}_{\zeta}^{-\nu(\alpha^2-2\alpha+\gamma)\gamma/2} \right] z \left[\mathbf{e}_{\zeta}^{\nu(\alpha^2-2\alpha+\gamma)\gamma(1-\alpha)w + \bar{w} - (1-\alpha/2)\zeta} \right] \left[\mathbf{e}_{\zeta}^{\nu\alpha\gamma} \right] d\zeta / (2\pi)^{d/2}$$

$$= z \left[\mathbf{e}_{\bar{z}}^{\nu(\alpha^2-2\alpha+\gamma)(1-\gamma)/2} \right] \frac{\gamma^2}{\alpha^2 - 2\alpha + \gamma} \int_{\mathbb{R}^d} \zeta \left[\mathbf{e}_{\zeta}^{-\nu\gamma^2/2(1-\alpha)w + \bar{w} + (\alpha-2+\gamma/\alpha)z} \right] \left[\mathbf{e}_{\zeta}^{\nu\alpha\gamma} \right] d\zeta / (2\pi)^{d/2}$$

$$= z \left[\mathbf{e}_{\bar{z}}^{\nu(\alpha^2-2\alpha+\gamma)(1-\gamma)/2} \right] (1-\alpha)w + \bar{w} + (\alpha-2+\gamma/\alpha)z \left[\mathbf{e}_{\left(1-\bar{\alpha}\right)\bar{w} + w + \frac{\bar{\alpha}-2+\bar{\gamma}/\bar{\alpha}}{\bar{\alpha}}\bar{z}}^{\nu\alpha^2/2} \right]$$

$$= \overbrace{\left[w \mathbf{e}_{\bar{w}}^{\alpha(1-\gamma)/2} w \mathbf{e}_w^{\gamma-\alpha} \bar{w} \mathbf{e}_w^{(1+\alpha)/2} \right]}^{\nu\alpha} w \left[\mathbf{e}_w^{-\nu(\alpha^2-2\alpha+\gamma)\alpha} \right] z \left[\mathbf{e}_w^{\nu(\alpha^2-2\alpha+\gamma)\alpha(1-\alpha)z + \alpha w} \right] \left[\mathbf{e}_{\left(1-\bar{\alpha}\right)\bar{z} + \bar{\alpha}\bar{w}}^{\nu(\alpha^2-2\alpha+\gamma)/2} \right]$$

$$= \overbrace{\left[w \mathbf{e}_{\bar{w}}^{\alpha(1-\gamma)/2} w \mathbf{e}_w^{\gamma-\alpha} \bar{w} \mathbf{e}_w^{(1+\alpha)/2} \right]}^{\nu\alpha} z \overbrace{\left[\frac{1-\alpha}{w} \right]}^{\nu(\alpha^2-2\alpha+\gamma)} I$$