

$$\mathbb{L}\mathbb{R}^d/\mathbb{C}^d \xleftarrow{\overline{(\cdot)}^{1-\alpha}} \mathbb{L}/\mathbb{R}^d \xrightarrow{\overline{(\cdot)}^{1-\alpha}} \mathbb{L}\mathbb{R}^d/\mathbb{C}^d$$

$\left\{ \begin{array}{l} \text{geod} \\ \text{Toep} \\ \text{Weyl} \end{array} \right.$

$$z \overline{x}^{1-\alpha\nu} = \frac{\overline{\nu\alpha/2}^{d/2}}{x \mathbf{e}_x^{\nu\alpha/2}} z \mathbf{e}_x^{\nu\alpha} z \mathbf{e}_{\bar{z}}^{\nu(1-\alpha)/2} \begin{array}{l} \text{Toep} \\ \text{Weyl} \end{array} \left\{ \begin{array}{ll} \overline{\nu/2}^{d/2} \frac{z \mathbf{e}_x^{2\nu}}{x \mathbf{e}_x^{\nu/2}} & \alpha = 1 \\ \frac{\nu^{d/2}}{x \mathbf{e}_x^\nu} z \mathbf{e}_x^{2\nu z} \mathbf{e}_{\bar{z}}^{-\nu/2} & \alpha = 2 \end{array} \right.$$

$$\overline{\gamma}^{1-\alpha\nu} = \int_{dx/\pi^{d/2}}^{d_{\mathbb{R}}} x \gamma \overline{x}^{1-\alpha\nu} \begin{array}{l} \text{Toep} \\ \text{Weyl} \end{array} \left\{ \begin{array}{ll} \int_{dx/\pi^{d/2}}^{d_{\mathbb{R}}} \frac{\overline{\nu/2}^{d/2}}{x \mathbf{e}_x^{\nu/2}} z \mathbf{e}_x^{2\nu} & \alpha = 1 \\ \int_{dx/\pi^{d/2}}^{d_{\mathbb{R}}} \frac{\nu^{d/2}}{x \mathbf{e}_x^\nu} z \mathbf{e}_x^{2\nu z} \mathbf{e}_{\bar{z}}^{-\nu/2} & \alpha = 2 \end{array} \right.$$

$$z \overline{\gamma}^{1-\alpha\nu} = \int_{dx/\pi^{d/2}}^{d_{\mathbb{R}}} x \gamma z \overline{x}^{1-\alpha\nu} = \int_{dx/\pi^{d/2}}^{d_{\mathbb{R}}} \frac{\overline{\nu\alpha/2}^{d/2}}{x \mathbf{e}_x^{\nu\alpha/2}} z \mathbf{e}_x^{\nu\alpha} z \mathbf{e}_{\bar{z}}^{\nu(1-\alpha)/2} x \gamma \begin{array}{l} \text{Toep} \\ \text{Weyl} \end{array} \left\{ \begin{array}{ll} \int_{dx/\pi^{d/2}}^{d_{\mathbb{R}}} \frac{\overline{\nu/2}^{d/2}}{x \mathbf{e}_x^{\nu/2}} z \mathbf{e}_x^{2\nu x} \gamma & \alpha = 1 \\ \int_{dx/\pi^{d/2}}^{d_{\mathbb{R}}} \frac{\nu^{d/2}}{x \mathbf{e}_x^\nu} z \mathbf{e}_x^{2\nu z} \mathbf{e}_{\bar{z}}^{-\nu/2 x} \gamma & \alpha = 2 \end{array} \right.$$

$$\int_{dx/\pi^d}^{\mathbb{R}^d} \frac{\det A^{1/2}}{x \mathbf{e}_x^A} z \mathbf{e}_\zeta^2 = z \mathbf{e}_{\bar{z}}^{A^{-1}}$$

$$z^{1-\alpha} \overline{z}^\nu = z \overline{z}^{\nu/2}$$

$$\begin{aligned} \text{LHS} &= \int_{dx/\pi^{d/2}}^{\mathbb{R}^d} z^{1-\alpha} \overline{z}^\nu = \int_{\mathbb{R}^d} \left(\frac{dx}{\sqrt{\pi}} \right)^d \frac{\nu \alpha / 2}{x \overline{z}^{\nu \alpha / 2}} z \overline{z}^{\nu \alpha} z \overline{z}^{\nu(1-\alpha)/2} = z \overline{z}^{\nu(1-\alpha)/2} \int_{dx/\pi^{d/2}}^{\mathbb{R}^d} \frac{\nu \alpha / 2}{x \overline{z}^{\nu \alpha / 2}} z \overline{z}^{\nu \alpha} \\ &= z \overline{z}^{\nu(1-\alpha)/2} \frac{\nu \alpha z / 2}{\nu \alpha \overline{z} / 2} = z \overline{z}^{\nu(1-\alpha)/2} z \overline{z}^{\nu \alpha / 2} = \text{RHS} \end{aligned}$$

$$z \overline{\mathfrak{R} | \gamma}_{\mathbb{R}^d} = z \mathfrak{R} | \gamma_{\mathbb{R}^d} = \int_{dx/\pi^{d/2}}^{\mathbb{R}^d} z \mathfrak{R}_x^x \gamma$$

$$z \mathfrak{R}_x = z \overline{e_x^{2\pi x}} e_x^{-\pi} z \overline{e_x^{-\pi/2}} = e^{z|x - z|\overline{z}/2 - x|x/2}$$

$$\mathbb{C}^n \xrightarrow[\varpi]{2} \bar{\mathbb{C}} \xleftarrow[\text{unitary}]{\mathcal{B}} \mathbb{R}^n \xrightarrow[\infty]{2} \mathbb{C}$$

$$\overline{\mathcal{W}_\nu \gamma} \star \overline{\mathcal{W}_\nu \acute{\gamma}} = \gamma \star \acute{\gamma}$$

$$\begin{aligned} \zeta \overline{\mathcal{B} \gamma} &= 2^{n/4} \int_{d\xi}^{\mathbb{R}^n} \xi \eta \zeta e_\xi^{2\pi} e_\xi^{-\pi} \zeta e_{\bar{\zeta}}^{-\pi/2} \\ \text{LHS} &= \int_{dz}^{\mathbb{C}^d} e^{-\nu z \star z} \overline{\mathcal{W}_\nu \gamma} \star \overline{\mathcal{W}_\nu \acute{\gamma}} \\ &= \int_{dz}^{\mathbb{C}^d} e^{-\nu z \star z} \int_{d\xi}^{\mathbb{R}^d} e^{\nu (2\xi \star z - \bar{z} \star z/2 - \xi \star \xi)} \xi \bar{\eta} \int_{d\eta}^{\mathbb{R}^d} e^{\nu (2z \star \eta - z \star \bar{z}/2 - \eta \star \eta)} \eta \acute{\gamma} \\ &= \int_{d\xi}^{\mathbb{R}^d} e^{-\nu \xi \star \xi} \xi \bar{\eta} \int_{d\eta}^{\mathbb{R}^d} e^{-\nu \eta \star \eta} \eta \acute{\gamma} \int_{dz}^{\mathbb{C}^d} e^{\nu (2\xi \star z + 2z \star \eta - \bar{z} \star z/2 - z \star \bar{z}/2 - z \star z)} \\ &= \int_{d\xi}^{\mathbb{R}^d} e^{-\nu \xi \star \xi} \xi \bar{\eta} \int_{d\eta}^{\mathbb{R}^d} e^{-\nu \eta \star \eta} \eta \acute{\gamma} \int_{dy}^{\mathbb{R}^d} e^{\nu (2iy \star (\eta - \xi))} \int_{dx/\pi^{d/2}}^{\mathbb{R}^d} e^{2\nu ((\xi + \eta) \star x - x \star x)} \\ &= \int_{d\zeta}^{\mathbb{R}^d} e^{-2\nu \zeta \star \zeta} \zeta \bar{\eta} \zeta \acute{\gamma} \int_{dx/\pi^{d/2}}^{\mathbb{R}^d} e^{2\nu (2\zeta \star x - x \star x)} = \int_{d\zeta}^{\mathbb{R}^d} e^{-2\nu \zeta \star \zeta} \zeta \bar{\eta} \zeta \acute{\gamma} e^{2\nu \zeta \frac{1}{2\nu} \star \zeta} = \int_{d\zeta}^{\mathbb{R}^d} \zeta \bar{\eta} \zeta \acute{\gamma} \end{aligned}$$

$$\mathbb{C}^n \xrightarrow[\pm]{2} \bar{\mathbb{C}} \xleftarrow[\text{even}]{\mathcal{B}} \mathbb{R}^n \xrightarrow[\pm]{2} \mathbb{C}$$