$$\left[\begin{array}{c|c} a & b \\ \hline b^* & c \end{array}\right] > 0 \Leftrightarrow \begin{cases} a > 0 \quad c > b^* a^{-1} b \Leftrightarrow \overline{\|c^{-\ 1/2} b^* a^{-\ 1/2}\|} < 1 \\ c > 0 \quad a > b c^{-1} b^* \Leftrightarrow \overline{\|a^{-\ 1/2} b c^{-\ 1/2}\|} < 1 \end{cases}$$

$$\left[\begin{array}{c|c} a & b \\ \hline b^* & c \end{array}\right] = \begin{cases} \left[\begin{array}{c|c} 1 & 0 \\ \hline b^* a^{-1} & 1 \end{array}\right] \left[\begin{array}{c|c} a & 0 \\ \hline 0 & c - b^* a^{-1} b \end{array}\right] \left[\begin{array}{c|c} 1 & a^{-1} b \\ \hline 0 & 1 \end{array}\right] \\ \left[\begin{array}{c|c} 1 & b c^{-1} \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c|c} a - b c^{-1} b^* & 0 \\ \hline 0 & c \end{array}\right] \left[\begin{array}{c|c} 1 & 0 \\ \hline c^{-1} b^* & 1 \end{array}\right] \end{cases}$$

$$\begin{cases} c > b^* a^{-1} b \Leftrightarrow I > c^{-\ 1/2} b^* a^{-\ 1/2} \overbrace{c^{-\ 1/2} b^* a^{-\ 1/2}}^{*} \Leftrightarrow \overline{\|c^{-\ 1/2} b^* a^{-\ 1/2}\|} < 1 \\ a > b c^{-1} b^* \Leftrightarrow I > a^{-\ 1/2} b c^{-\ 1/2} \overbrace{a^{-\ 1/2} b c^{-\ 1/2}}^{*} \Leftrightarrow \overline{\|a^{-\ 1/2} b c^{-\ 1/2}\|} < 1 \end{cases}$$

$$\left[\begin{array}{c|c} x & y \end{array}\right] \left[\begin{array}{c|c} 1 & 1 \\ \hline i & -i \end{array}\right] \left[\begin{array}{c|c} a & b \\ \hline b^+ & d \end{array}\right] \left[\begin{array}{c|c} 1 & 1 \\ \hline -i & i \end{array}\right] \left[\begin{array}{c} x^+ \\ \hline y^+ \end{array}\right] = \left[\begin{array}{c|c} z & \bar{z} \end{array}\right] \left[\begin{array}{c|c} a & b \\ \hline b^+ & d \end{array}\right] \left[\begin{array}{c} z^+ \\ \hline z^* \end{array}\right]$$

$$= \left[\begin{array}{c|c} z & \bar{z} \end{array}\right] \left[\begin{array}{c|c} a & b \\ \hline b^+ & d \end{array}\right] \left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array}\right] \left[\begin{array}{c} z^* \\ \hline z^+ \end{array}\right] = \left[\begin{array}{c|c} z & \bar{z} \end{array}\right] \left[\begin{array}{c|c} b & a \\ \hline d & b^+ \end{array}\right] \left[\begin{array}{c} z^* \\ \hline z^+ \end{array}\right]$$

$$\left[\begin{array}{c|c} b & a \\ \hline d & b^+ \end{array}\right] + \left[\begin{array}{c|c} b & a \\ \hline d & b^+ \end{array}\right]^{*} = \left[\begin{array}{c|c} b + b^* & a + \bar{d} \\ \hline d + \bar{a} & b^+ + \bar{b} \end{array}\right] > 0 \Leftrightarrow$$

$$\begin{cases} b + b^* > 0 \\ b^+ + \bar{b} > \underline{\bar{a} + d} \overbrace{b + b^*}^{-1} \underline{a + \bar{d}} \Leftrightarrow \overline{\|\underbrace{b^+ + \bar{b}}_{-\ 1/2} \underline{\bar{a} + d} \underbrace{b + b^*}_{-\ 1/2}\|} < 1 \end{cases}$$

$$\left[\begin{array}{c|c} a & b \\ \hline b^+ & d \end{array}\right]^{-1} = \left[\begin{array}{c|c} -b^{+-1} d \overbrace{b - a b^{+-1} d}^{-1} & \overbrace{b^+ - d b^{-1} a}^{-1} \\ \hline \underbrace{b - a b^{+-1} d}_{-1} & -b^{-1} a \underbrace{b^+ - d b^{-1} a}_{-1} \end{array}\right]$$

$$2\!\!\!\!/\,^d\pi \int\limits_{dz}^{\mathbb{C}^d} {}^z\mathcal{E}_z^{-a}\,{}^z\mathcal{E}_{\bar z}^{-b}\,{}^{\bar z}\mathcal{E}_z^{-c}\,{}^{\bar z}\mathcal{E}_{\bar z}^{-a^+}\,{}^w\mathcal{E}_z\,{}^z\mathcal{E}_\zeta = 2\!\!\!\!/\,^d\pi \int\limits_{dz}^{\mathbb{C}^d} {}^{z|\bar z}\boxed{\mathcal{E}}^{-\begin{smallmatrix}a & b\\ b^+ & d\end{smallmatrix}}_{\overline z|z}\,{}^{u|v}_{\;2}\mathcal{E}_{\overline z|z} = \overbrace{\begin{smallmatrix}-a & -b\\ + & \\ b & d\end{smallmatrix}}^{-1/2}\,{}^{u|v}\mathcal{E}^{\begin{smallmatrix}a & -1 & b\\ b^+ & & d\end{smallmatrix}}_{\overline u|\overline v}$$

$$= \overset{-1/2}{\overleftarrow{b}}\,\overbrace{b-a\overset{+-1}{b}d}^{-1/2}\ \exp\ u\mid v\ \frac{-\overset{+-1}{b}d\,\overbrace{b-a\overset{+-1}{b}d}^{-1}\ \Big|\ \overbrace{\overset{+}{b}-db^{-1}a}^{-1}}{\underbrace{b-a\overset{+-1}{b}d}_{-1}\ \Big|\ -b^{-1}a\underbrace{\overset{+}{b}-db^{-1}a}_{-1}}\ \frac{\overset{+}{u}}{\overset{+}{v}}$$

$$\left(\frac{2\overset{d}{\nu}}{\pi}\right)\int\limits_{dz}^{\mathbb{C}^d} {}^{z|\bar z}_{\;\nu}\mathcal{E}^{-\begin{smallmatrix}a & b\\ b^+ & d\end{smallmatrix}}_{\overline z|z}\,{}^u_{2\nu}\mathcal{E}_{\overline z}\,{}^v_{2\nu}\mathcal{E}_z = \overbrace{\begin{smallmatrix}-a & -b\\ b^+ & d\end{smallmatrix}}^{-1/2}\,{}^{u|v}_{\;\nu}\mathcal{E}^{\begin{smallmatrix}a & -1 & b\\ b^+ & & d\end{smallmatrix}}_{\overline u|\overline v}$$

$$= \overset{-1/2}{\overleftarrow{b}}\,\overbrace{b-a\overset{+-1}{b}d}^{-1/2}\ \exp\nu\ u\mid v\ \frac{-\overset{+-1}{b}d\,\overbrace{b-a\overset{+-1}{b}d}^{-1}\ \Big|\ \overbrace{\overset{+}{b}-db^{-1}a}^{-1}}{\underbrace{b-a\overset{+-1}{b}d}_{-1}\ \Big|\ -b^{-1}a\underbrace{\overset{+}{b}-db^{-1}a}_{-1}}\ \frac{\overset{+}{u}}{\overset{+}{v}}$$

$$\int\limits_{dz}^{\mathbb{C}^d}\exp-\begin{bmatrix}z & \bar z\end{bmatrix}\frac{a\ \big|\ b}{c\ \big|\ a^+}\,\frac{\overset{*}{z}}{\overset{+}{z}}\exp 2\begin{bmatrix}w & \bar w\end{bmatrix}\frac{\overset{*}{z}}{\overset{+}{z}} = \pi\!\!\!\!/\,^d 2\,\overbrace{\frac{a\ \big|\ b}{c\ \big|\ a^+}}^{-1/2}\ \exp\begin{bmatrix}w & \bar w\end{bmatrix}\frac{a\ \big|^{-1}\ b}{c\ \big|\ a^+}\,\frac{\overset{*}{w}}{\overset{+}{w}}$$

$$Q = \frac{1\ \big|\ 1}{i\ \big|\ -i}\,\frac{a\ \big|\ b}{c\ \big|\ a^+}\,\frac{1\ \big|\ -i}{1\ \big|\ i}\ \Rightarrow\ \overset{1/2}{\overleftarrow{Q}} = 2^d\,\overbrace{\frac{a\ \big|\ b}{c\ \big|\ a^+}}^{1/2}$$

$$\overset{-1}{\overline Q} = \frac{1\ \big|^{-1}\ -i}{1\ \big|\ i}\,\frac{a\ \big|^{-1}\ b}{c\ \big|\ a^+}\,\frac{1\ \big|^{-1}\ 1}{i\ \big|\ -i} = \frac{1}{4}\,\frac{1\ \big|\ 1}{i\ \big|\ -i}\,\frac{\alpha\ \big|\ \beta}{\gamma\ \big|\ \alpha^+}\,\frac{1\ \big|\ -i}{1\ \big|\ i}$$

$$\begin{bmatrix}z & \bar z\end{bmatrix}\frac{a\ \big|\ b}{c\ \big|\ a^+}\,\frac{\overset{*}{z}}{\overset{+}{z}} = \underbrace{\begin{bmatrix}x & y\end{bmatrix}\frac{1\ \big|\ 1}{i\ \big|\ -i}}\,\frac{a\ \big|\ b}{c\ \big|\ a^+}\,\overbrace{\begin{bmatrix}x & y\end{bmatrix}\frac{1\ \big|\ 1}{i\ \big|\ -i}}^{*} = \begin{bmatrix}x & y\end{bmatrix}Q\frac{\overset{*}{x}}{\overset{*}{y}}$$

$$2\begin{bmatrix}w & \bar w\end{bmatrix}\frac{\overset{*}{z}}{\overset{+}{z}} = 2\underbrace{\begin{bmatrix}u & v\end{bmatrix}\frac{1\ \big|\ 1}{i\ \big|\ -i}}\,\overbrace{\begin{bmatrix}x & y\end{bmatrix}\frac{1\ \big|\ 1}{i\ \big|\ -i}}^{*} = 4\begin{bmatrix}u & v\end{bmatrix}\frac{\overset{*}{x}}{\overset{*}{y}}$$

$$4\begin{bmatrix}u & v\end{bmatrix}\overset{-1}{\overline Q}\frac{\overset{+}{u}}{\overset{+}{v}} = \underbrace{\begin{bmatrix}u & v\end{bmatrix}\frac{1\ \big|\ 1}{i\ \big|\ -i}}\,\frac{a\ \big|^{-1}\ b}{c\ \big|\ a^+}\,\overbrace{\begin{bmatrix}u & v\end{bmatrix}\frac{1\ \big|\ 1}{-i\ \big|\ i}}^{+} = \begin{bmatrix}w & \bar w\end{bmatrix}\frac{a\ \big|^{-1}\ b}{c\ \big|\ a^+}\,\frac{\overset{*}{w}}{\overset{+}{w}}$$

$$\text{LHS} = \int\limits_{dx}^{\mathbb{R}^d}\int\limits_{dy}^{\mathbb{R}^d}\exp-\begin{bmatrix}x & y\end{bmatrix}Q\frac{\overset{*}{x}}{\overset{*}{y}}\exp 4\begin{bmatrix}u & v\end{bmatrix}\frac{\overset{*}{x}}{\overset{*}{y}} = \pi^d\,\overset{-1/2}{\overleftarrow{Q}}\exp\begin{bmatrix}2u & 2v\end{bmatrix}\overset{-1}{\overline Q}\frac{2}{2}\frac{\overset{+}{u}}{\overset{+}{v}} = \text{RHS}$$