

$$\mathbb{L} \otimes_{\mathbb{R}} \overset{2}{\mathbb{C}}$$

\downarrow $\left(\begin{array}{c} \text{quant} / \text{Geod} / \text{Toep} / \text{Weyl} \\ \text{I} - \alpha \end{array} \right)$

$$\mathbb{L}_{\Delta_w}^2 \overset{\nu}{\mathbb{C}} \overset{2}{\mathbb{C}} \overset{2}{\mathbb{C}} = \mathbb{L}_{\Delta_w}^2 \overset{\nu}{\mathbb{C}} \overset{\sharp}{\mathbb{C}}$$

$$s_0^\alpha z = \alpha z$$

$$s_w^\alpha z = s_{g_w 0}^\alpha z = g_w s_0^\alpha g_w^{-1} z = w + \alpha(z - w) = \alpha z + (1 - \alpha)w$$

$$\overset{z}{\mathbb{J}} \overset{\nu}{\mathbb{C}} = \nu \int_{d^2 w / \pi}^{\mathbb{C}} \nu^{(1-\alpha)(z-w)} \bar{w} e_w \mathbb{J}^{\alpha z + (1-\alpha)w} \gamma$$

$$\mathbb{J} = \nu \int_{d^2 w / \pi}^{\mathbb{C}} w \mathbb{J}^w \bar{w}$$

$$\alpha = 0 \Rightarrow \text{Toeplitz } \overset{z}{\mathbb{J}} \overset{\nu}{\mathbb{C}} = \nu \int_{d^2 w / \pi}^{\mathbb{C}} \nu^{(z-w)} \bar{w} e_w \mathbb{J}^w \gamma = \nu \int_{d^2 w / \pi}^{\mathbb{C}} -\nu w \bar{w} e^{\nu z \bar{w}} e_w \mathbb{J}^w \gamma$$

$$\alpha = -1 \Rightarrow \text{Weyl } \overset{z}{\mathbb{J}} \overset{\nu}{\mathbb{C}} = \nu \int_{d^2 w / \pi}^{\mathbb{C}} 2\nu(z-w) \bar{w} e_w \mathbb{J}^{2w-z} \gamma$$

$$\overline{\nu\alpha}^d \int_{dx/\pi^d}^{\mathbb{C}^d} \frac{1-\alpha}{x} \stackrel{\text{normalize}}{=} I$$

$$\overline{\nu\alpha}^d \int_{dx/\pi^d}^{\mathbb{C}^d} z \overline{\frac{1-\alpha}{x}} \eta = \overline{\nu\alpha}^d \int_{dx/\pi^d}^{\mathbb{C}^d} (1-\alpha)z + \alpha x \eta z \mathcal{E}_x^{\nu\alpha} x \mathcal{E}_x^{-\nu\alpha} = \int_{dx/\pi^d}^{\mathbb{C}^d} \frac{\overline{\nu\alpha}^d}{x \mathcal{E}_x^{\nu\alpha}} z \mathcal{E}_x^{\nu\alpha} (1-\alpha)z + \alpha x \eta = (1-\alpha)z + \alpha z \eta = z \eta$$

$$\overline{\frac{1-\alpha}{J}} = \overline{\frac{d}{\nu\alpha}} \int_{dw/\pi^d}^{\mathbb{C}^d} w \overline{\frac{1-\alpha}{w}}$$

$$z \overline{\frac{1-\alpha}{J}} = \int_{dw/\pi^d}^{\mathbb{C}^d} \frac{(\nu\alpha)^d}{w e_w^{\nu\alpha}} z \mathcal{E}_w^{\nu\alpha} w \overline{\frac{1-\alpha}{w}} \begin{cases} \int_{dw/\pi^d}^{\mathbb{C}^d} \frac{\nu^d}{w e_w^\nu} z \mathcal{E}_w^\nu w \overline{\frac{1-\alpha}{w}} \eta & \alpha = 1 \\ \int_{dw/\pi^d}^{\mathbb{C}^d} \frac{(2\nu)^d}{w e_w^{2\nu}} z \mathcal{E}_w^{2\nu} w \overline{\frac{1-\alpha}{w}} \eta & \alpha = 2 \end{cases}$$

$$z \overline{\frac{J \neq \mathfrak{J}}{Weyl}} = \left(\frac{d}{\pi}\right) \int_{dx}^{\mathbb{C}_d} x \overline{\frac{J \neq \mathfrak{J}}{Weyl}} \int_{dy}^{\mathbb{C}_d} y \overline{\frac{J \neq \mathfrak{J}}{Weyl}} \underbrace{\begin{matrix} x|y|z & 0 & 1 & -1 \\ \mathcal{E} & -1 & 0 & 1 \\ & 1 & -1 & 0 \\ & x|y|z & & \end{matrix}}_{= z^{-x} \mathcal{E}_{z-y} \quad z^{-y} \mathcal{E}_{z-x}^{-1}}$$

$$\zeta \overline{\frac{J \neq \mathfrak{J}}{Up}} \stackrel{\text{Ar}}{=} \left(\frac{\nu(1-\alpha)(1-\beta)(1-\gamma)}{2\pi(\alpha\beta-\gamma)}\right) \int_{d\xi}^{\mathbb{C}_d} \xi \overline{\frac{J \neq \mathfrak{J}}{Up}} \int_{d\eta}^{\mathbb{C}_d} \eta \overline{\frac{J \neq \mathfrak{J}}{Up}} \underbrace{\begin{matrix} \xi|\eta|\zeta & -a & -b & a+b \\ \mathcal{E} & -c & -d & c+d \\ & a+c & b+d & -a-b-c-d \\ & \xi|\eta|\zeta & & \end{matrix}}_{= \zeta^{-\xi} \mathcal{E}_{\zeta-\xi}^a \quad \zeta^{-\xi} \mathcal{E}_{\zeta-\eta}^b \quad \zeta^{-\eta} \mathcal{E}_{\zeta-\xi}^c \quad \zeta^{-\eta} \mathcal{E}_{\zeta-\eta}^d}$$

$$\underbrace{e_\alpha \mathfrak{K} \mathfrak{J}}_z \neq \underbrace{e_\beta \mathfrak{K} \mathfrak{J}}_z = \left(\frac{\nu(1-\alpha)(1-\beta)}{\pi(1+\alpha\beta)} \right) \int_{d\xi}^{\mathbb{C}_d} \int_{d\eta}^{\mathbb{C}_d} \eta \mathfrak{J} \nu/1+\alpha\beta \mathcal{E} \begin{array}{c|c|c} \xi|\eta|z & \frac{-(1-\alpha)(1+\beta)}{2(1-\alpha)} & \frac{(1-\alpha)(1-\beta)}{2\alpha(1-\beta)} & \frac{2(1-\alpha)\beta}{-2(1-\alpha\beta)} \\ \hline & & & \end{array}$$

$$\begin{aligned} \text{LHS} &= \left(\frac{2\nu}{\pi} \right) \int_{dx}^{\mathbb{C}_d} \int_{dy}^{\mathbb{C}_d} x \underbrace{e_\alpha \mathfrak{K} \mathfrak{J}}_y \underbrace{e_\beta \mathfrak{K} \mathfrak{J}}_y \begin{array}{c|c|c} x|y|z & \frac{0}{-1} & \frac{1}{0} & \frac{-1}{1} \\ \hline & & & \end{array} \\ &= \underbrace{\left(\frac{2\nu}{\pi} \right) \int_{d\xi}^{\mathbb{C}_d} \int_{d\eta}^{\mathbb{C}_d} \eta \mathfrak{J}}_{=*} \left(\frac{2\nu\tilde{\alpha}}{\pi} \right) \left(\frac{2\nu\tilde{\beta}}{\pi} \right) \int_{dx}^{\mathbb{C}_d} \int_{dy}^{\mathbb{C}_d} x^{-\xi} \mathcal{E} \begin{array}{c|c|c} x-\xi & \frac{0}{-1} & \frac{1}{-1} & \frac{-1}{0} \\ \hline & & & \end{array} \\ &= * \left(\tilde{\alpha}\tilde{\beta} \right)^d \frac{\xi}{2\nu} \mathcal{E} \begin{array}{c|c|c} \xi & \frac{\tilde{\alpha}}{1} & \frac{-1}{\tilde{\beta}} \\ \hline & & \end{array} \frac{\eta}{2\nu} \mathcal{E} \begin{array}{c|c|c} \eta & \frac{0}{-1} & \frac{0}{\tilde{\beta}} \\ \hline & & \end{array} \xi|\eta|z \mathcal{E} \begin{array}{c|c|c} \xi|\eta|z & \frac{\tilde{\alpha}}{1} & \frac{0}{-1} \\ \hline & & \end{array} \\ &\stackrel{\text{Gauss}}{=} * \left(\tilde{\alpha}\tilde{\beta} \right)^d \frac{\xi}{2\nu} \mathcal{E} \begin{array}{c|c|c} \xi & \frac{-1}{1} & \frac{0}{\tilde{\beta}} \\ \hline & & \end{array} \frac{\eta}{2\nu} \mathcal{E} \begin{array}{c|c|c} \eta & \frac{0}{-1} & \frac{-1}{\tilde{\beta}} \\ \hline & & \end{array} \xi|\eta|z \mathcal{E} \begin{array}{c|c|c} \xi|\eta|z & \frac{\tilde{\alpha}}{1} & \frac{0}{-1} \\ \hline & & \end{array} \\ &= \left(1 + \tilde{\alpha}\tilde{\beta} \right)^d \\ &= * \left(\frac{2\nu\tilde{\alpha}\tilde{\beta}}{1 + \tilde{\alpha}\tilde{\beta}} \right) \frac{\xi|\eta|z}{2\nu/1+\tilde{\alpha}\tilde{\beta}} \mathcal{E} \begin{array}{c|c|c} \xi|\eta|z & \frac{\tilde{\alpha}^2\tilde{\beta} - \tilde{\alpha}(1+\tilde{\alpha}\tilde{\beta})}{-\tilde{\alpha}\tilde{\beta}} & \frac{\tilde{\alpha}\tilde{\beta}}{\tilde{\alpha}\tilde{\beta}^2 - \tilde{\beta}(1+\tilde{\alpha}\tilde{\beta})} & \frac{\tilde{\alpha}(1-\tilde{\beta})}{(1+\tilde{\alpha})\tilde{\beta}} \\ \hline & & & \end{array} = \text{RHS} \\ &\frac{\tilde{\alpha}}{1} \begin{array}{c|c} -1 \\ \hline \tilde{\beta} \end{array} = \frac{1}{1 + \tilde{\alpha}\tilde{\beta}} \begin{array}{c|c} \tilde{\beta} \\ \hline -1 \end{array} \begin{array}{c|c} 1 \\ \hline \tilde{\alpha} \end{array} : 1 + \tilde{\alpha}\tilde{\beta} = \frac{2(1+\alpha\beta)}{(1+\alpha)(1+\beta)} \end{aligned}$$

$$N = \tilde{\gamma} + a + b + c + d$$

$$z \underbrace{e_\gamma \bowtie \mathbb{J}_\# \mathbb{J}} = \left(\frac{\nu (1-\alpha) (1-\beta) (1-\gamma) \tilde{\gamma}}{2\pi (\alpha\beta - \gamma) N} \right) \int_{d\xi}^{\mathbb{C}_d} \int_{d\eta}^{\mathbb{C}_d} \eta \mathbb{J} \int_{2\nu/N}^{\xi|\eta|z} \mathcal{E} \left[\begin{array}{c|c|c} bc-ad-\tilde{\gamma}a & ad-bc-\tilde{\gamma}b & (a+b)\tilde{\gamma} \\ \hline ad-bc-\tilde{\gamma}c & bc-ad-\tilde{\gamma}d & (c+d)\tilde{\gamma} \\ \hline \tilde{\gamma}(a+c) & \tilde{\gamma}(b+d) & -\tilde{\gamma}(a+b+c+d) \end{array} \right]_{\xi|\eta|z}$$

$$\text{LHS} = \int_{\mathbb{C}^d} z^{-\zeta} e_\gamma \int_{\zeta} \mathbb{J}_\# \mathbb{J} = C \int_{\mathbb{C}^d} z^{-\zeta} e_\gamma \int_{d\xi}^{\mathbb{C}_d} \int_{d\eta}^{\mathbb{C}_d} \eta \mathbb{J} \int_{2\nu}^{\xi|\eta|\zeta} \mathcal{E} \left[\begin{array}{c|c|c} -a & -b & a+b \\ \hline -c & -d & c+d \\ \hline a+c & b+d & -a-b-c-d \end{array} \right]_{\xi|\eta|\zeta}$$

$$= C \underbrace{\int_{d\xi}^{\mathbb{C}_d} \int_{d\eta}^{\mathbb{C}_d} \eta \mathbb{J}}_{=*} \left(\frac{2\nu\tilde{\gamma}}{\pi} \right) \int_{\mathbb{C}^d} z^{-\zeta} \mathcal{E}_{z-\zeta}^{-\tilde{\gamma}} \int_{2\nu}^{\xi|\eta|\zeta} \mathcal{E} \left[\begin{array}{c|c|c} -a & -b & a+b \\ \hline -c & -d & c+d \\ \hline a+c & b+d & -a-b-c-d \end{array} \right]_{\xi|\eta|\zeta}$$

$$= * \tilde{\gamma}^d \int_{\mathbb{C}^d} z \mathcal{E}_z^{-\tilde{\gamma}} \int_{-2\nu}^{\xi|\eta} \mathcal{E} \left[\begin{array}{c|c} a & b \\ \hline c & d \end{array} \right]_{\xi|\eta} \left(\frac{2\nu}{\pi} \right) \int_{\mathbb{C}^d} \zeta \mathcal{E}_\zeta^{-a-b-c-d-\tilde{\gamma}} \int_{2\nu}^{\xi|\eta|\zeta} \mathcal{E} \left[\begin{array}{c|c} a+c & b+d \\ \hline \tilde{\gamma} & \tilde{\gamma} \end{array} \right]_{\xi|\eta|\zeta} \int_{2\nu}^{\xi|\eta|z} \mathcal{E} \left[\begin{array}{c} a+b \\ \hline c+d \\ \hline \tilde{\gamma} \end{array} \right]_{\zeta}$$

$$\stackrel{\text{Gauss}}{=} * \tilde{\gamma}^d N^{-d} \int_{\mathbb{C}^d} z \mathcal{E}_z^{-\tilde{\gamma}} \int_{-2\nu}^{\xi|\eta} \mathcal{E} \left[\begin{array}{c|c} a & b \\ \hline c & d \end{array} \right]_{\xi|\eta} \int_{2\nu/N}^{\xi|\eta|z} \mathcal{E} \left[\begin{array}{c|c} a+b & c+d \\ \hline \tilde{\gamma} & \tilde{\gamma} \end{array} \right]_{\xi|\eta|z} a+c \mid b+d \mid \tilde{\gamma}$$

$$= * \left(\frac{\tilde{\gamma}}{N} \right)^d \int_{2\nu/N}^{\xi|\eta|z} \mathcal{E} \left[\begin{array}{c|c|c} (a+b)(a+c)-aN & (a+b)(b+d)-bN & (a+b)\tilde{\gamma} \\ \hline (c+d)(a+c)-cN & (c+d)(b+d)-dN & (c+d)\tilde{\gamma} \\ \hline \tilde{\gamma}(a+c) & \tilde{\gamma}(b+d) & \tilde{\gamma}^2 - \tilde{\gamma}N \end{array} \right]_{\xi|\eta|z} = \text{RHS}$$

$$\underbrace{e_\alpha \bowtie J} \neq \underbrace{e_\beta \bowtie J} = e_\gamma \bowtie \underbrace{J \# J}$$

$$2(\alpha\beta - \gamma) \frac{a \mid b}{c \mid d} = \frac{(1-\alpha)(\beta-\gamma) \mid (1-\alpha)(1-\beta)\gamma}{(1-\alpha)(1-\beta) \mid (\alpha-\gamma)(1-\beta)}$$

$$\frac{2(\alpha\beta - \gamma)}{1-\gamma} (a+b+c+d) = 1 - \alpha\beta$$

$$2(\alpha\beta - \gamma) N = 2(\alpha\beta - \gamma) (a+b+c+d+\tilde{\gamma}) = \frac{(1-\gamma)^2 (1+\alpha\beta)}{1+\gamma} = \tilde{\gamma} (1-\gamma) (1+\alpha\beta)$$

$$1 + \tilde{\alpha} \tilde{\beta} = \frac{2(1+\alpha\beta)}{(1+\alpha)(1+\beta)} \Rightarrow \frac{4(\alpha\beta - \gamma)}{\tilde{\gamma}} \frac{N}{1+\tilde{\alpha}\tilde{\beta}} = (1+\alpha)(1+\beta)(1-\gamma)$$

$$4(\alpha\beta - \gamma)(ad - bc) = (1-\alpha)(1-\beta)(1-\gamma) \Rightarrow \frac{4(\alpha\beta - \gamma)}{\tilde{\gamma}} (ad - bc) = (1-\alpha)(1-\beta)(1+\gamma)$$

$$\frac{4(\alpha\beta - \gamma)}{\tilde{\gamma}} \frac{bc - ad - \tilde{\gamma}a \mid ad - bc - \tilde{\gamma}b}{ad - bc - \tilde{\gamma}c \mid bc - ad - \tilde{\gamma}d} = \frac{(1-\gamma)}{\tilde{\gamma}} \frac{N/2}{1+\alpha\beta} \frac{-(1-\alpha)(1+\beta) \mid (1-\alpha)(1-\beta)}{-(1-\alpha)(1-\beta) \mid -(1+\alpha)(1-\beta)} \Rightarrow$$

$$\frac{\frac{bc - ad - \tilde{\gamma}a \mid ad - bc - \tilde{\gamma}b}{ad - bc - \tilde{\gamma}c \mid bc - ad - \tilde{\gamma}d}}{\tilde{\gamma}(a+c) \mid \tilde{\gamma}(b+d)} \frac{(a+b)\tilde{\gamma} \mid (c+d)\tilde{\gamma}}{-\tilde{\gamma}(a+b+c+d)} = \frac{N/2}{1+\alpha\beta} \frac{-(1-\alpha)(1+\beta) \mid (1-\alpha)(1-\beta)}{2(1-\alpha) \mid 2\alpha(1-\beta)} \frac{2(1-\alpha)\beta \mid 2(1-\beta)}{-2(1-\alpha\beta)}$$

$$\left(\frac{2\nu}{\pi} \right) \left(\frac{\tilde{\alpha}\tilde{\beta}}{1+\tilde{\alpha}\tilde{\beta}} \right) = \left(\frac{\nu(1-\alpha)(1-\beta)(1-\gamma)\tilde{\gamma}}{2\pi(\alpha\beta - \gamma)N} \right) \Rightarrow {}_z\text{LHS} = {}_z\text{RHS}$$

$$\begin{aligned}
\overline{\mathfrak{t}_\xi^\nu \eta} \overline{\mathfrak{t}_\zeta^{1-\alpha} \mathfrak{t}_\eta^\nu} &= \nu^d \int_{dz/\pi^d} z \mathcal{E}_z^{-\nu} z \overline{\mathfrak{t}_\xi^\nu \eta} \overline{\mathfrak{t}_\zeta^{1-\alpha} \mathfrak{t}_\eta^\nu} = \nu^d \int_{dz/\pi^d} z \mathcal{E}_z^{-\nu} z + \xi \bar{\eta} \xi \mathcal{E}_z^{-\nu} \xi \mathcal{E}_\xi^{-\nu/2} (1-\alpha)z + \alpha\zeta \overline{\mathfrak{t}_\eta^\nu} z \mathcal{E}_\zeta^{\nu\alpha} z \mathcal{E}_z^{-\nu\alpha} \\
&= \nu^d \int_{dz/\pi^d} z \mathcal{E}_z^{\nu(-1-\alpha)} z + \xi \bar{\eta} \xi \mathcal{E}_z^{-\nu} \xi \mathcal{E}_\xi^{-\nu/2} (1-\alpha)z + \alpha\zeta + \eta \mathfrak{t} (1-\alpha)z + \alpha\zeta \mathcal{E}_\eta^{-\nu} \eta \mathcal{E}_\eta^{-\nu/2} z \mathcal{E}_\zeta^{\nu\alpha} \\
\stackrel{w=z+\xi}{=} \nu^d \int_{dw/\pi^d} w - \xi \mathcal{E}_w^{\nu(-1-\alpha)} w \bar{\eta} \xi \mathcal{E}_w^{-\nu} \xi \mathcal{E}_\xi^{-\nu/2} (1-\alpha)(w-\xi) + \alpha\zeta + \eta \mathfrak{t} (1-\alpha)(w-\xi) + \alpha\zeta \mathcal{E}_\eta^{-\nu} \eta \mathcal{E}_\eta^{-\nu/2} w - \xi \mathcal{E}_\zeta^{\nu\alpha}
\end{aligned}$$

$$\overline{\mathfrak{t}_\nu^\nu} \overline{\mathfrak{t}_\vartheta^{1-\alpha}} = \nu^d \int_{dw/\pi^d} w \mathcal{E}_w^{\nu(-1-\alpha)} w \bar{\eta}^{\alpha w + \alpha\vartheta} \mathfrak{t} w \mathcal{E}_\vartheta^{\nu\alpha}$$

$$\vartheta = \zeta + \frac{\eta - \alpha\xi}{1 - \alpha}$$

$$x \mid y \mid z \begin{array}{c|c|c} a & b & -a-b \\ \hline c & d & -c-d \\ \hline -a-c & -b-d & a+b+c+d \end{array} \begin{array}{c} \bar{x} \\ \bar{y} \\ \bar{z} \end{array} = x - z \mid y - z \begin{array}{c|c} a & b \\ \hline c & d \end{array} \begin{array}{c} \bar{x} - \bar{z} \\ \bar{y} - \bar{z} \end{array}$$

$$\begin{aligned}
\underline{f \# g} &= {}_x f {}_y g \mathcal{E}_{x-z}^{a} \mathcal{E}_{y-z}^{b} \mathcal{E}_{x-z}^{c} \mathcal{E}_{y-z}^{d} \\
&= \mathcal{E}_z^{-a-b} \mathcal{E}_x^{-a-c} z \mathcal{E}_z^{a+b+c+d} x \mathcal{E}_y^b z \mathcal{E}_y^{-b-d} y \mathcal{E}_x^c y \mathcal{E}_z^{-c-d} x \mathcal{E}_x^a f y \mathcal{E}_y^d g
\end{aligned}$$

$$\overline{f \# g}^\zeta = \zeta \mathcal{E}_z^z \mathcal{E}_\zeta^{-1} \underline{f \# g} = \zeta \mathcal{E}_z^z \mathcal{E}_\zeta^{-1} x \mathcal{E}_z^{-a-b} z \mathcal{E}_x^{-a-c} z \mathcal{E}_z^{a+b+c+d} x \mathcal{E}_y^b z \mathcal{E}_y^{-b-d} y \mathcal{E}_x^c y \mathcal{E}_z^{-c-d} x \mathcal{E}_x^a y \mathcal{E}_y^d f y g$$

$$\overline{\mathfrak{J}}^{1-\alpha} = \overline{\Lambda \mathfrak{J}}_{1-\alpha}$$

$$\Lambda \mathfrak{J} \in \mathbb{Z} \begin{array}{c} \mathbb{S} \\ \mathbb{Z} \end{array} \mathbb{C} \Rightarrow \overline{\mathfrak{J}}^{1-\alpha} = \overline{\Lambda \mathfrak{J}}_{1-\alpha} \in \mathbb{U} \mid \mathbb{Z} \begin{array}{c} \mathbb{Z} \\ \mathbb{Z} \end{array} \mathbb{C}$$

$$\overline{\mathfrak{J}}^{1-\alpha} \overline{\mathfrak{J}}^\beta = \overline{\Lambda \mathfrak{J}}_{1-\alpha} \overline{\Lambda \mathfrak{J}}_{1-\beta} = \overline{\Lambda \mathfrak{J} \# \Lambda \mathfrak{J}}_{1-\alpha} \overline{\Lambda \mathfrak{J}}_{1-\beta}$$

$$x_{1-\alpha}^{\Lambda y} = \underbrace{x}_{1-\alpha}^{\overline{y}}^{1-\alpha} = \text{tr } \mathfrak{s}_x^\nu \overline{y}^{1-\alpha}$$

$$\overline{\Lambda J}^{1-\alpha} = \overline{J}^{1-\alpha} = \overline{\overline{J}}^{1-\alpha} \Rightarrow \nu^d \int_{dy} x_{1-\alpha}^{\Lambda y} y^J = \underbrace{x}_{1-\alpha}^{\overline{\Lambda J}} = \underbrace{x}_{1-\alpha}^{\overline{J}}^{1-\alpha} = \nu^d \int_x \overline{y}^{1-\alpha} y^J$$

$$I = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} \underbrace{\mathcal{E}_z^{\nu z} \mathcal{E}_z^{-\nu/2}} \overbrace{\mathcal{E}_z^{\nu z} \mathcal{E}_z^{-\nu/2}}^* = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} z \mathcal{E}_z^{-\nu} \mathcal{E}_z^\nu \mathcal{E}_z^{*\nu}$$

$$\mathfrak{I} \overline{\mathfrak{I}} \mathfrak{I} = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} z \mathcal{E}_z^{-\nu} z \overline{\mathfrak{I}} z \mathfrak{I} = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} z \mathcal{E}_z^{-\nu} \mathcal{E}_z^\nu \overline{\mathfrak{I}} \mathfrak{I} \mathcal{E}_z^\nu \overline{\mathfrak{I}} \mathfrak{I} = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} z \mathcal{E}_z^{-\nu} \mathfrak{I} \overline{\mathfrak{I}} \mathcal{E}_z^\nu \mathcal{E}_z^\nu \overline{\mathfrak{I}} \mathfrak{I}$$

$$= \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} \mathfrak{I} \overline{\mathfrak{I}} \underbrace{\mathcal{E}_z^{\nu z} \mathcal{E}_z^{-\nu/2}} \underbrace{\mathcal{E}_z^{\nu z} \mathcal{E}_z^{-\nu/2}} \overline{\mathfrak{I}} \mathfrak{I} = \nu^d \mathfrak{I} \overline{\mathfrak{I}} \underbrace{\int_{dz/\pi^d}^{\mathbb{C}^d} \mathcal{E}_z^{\nu z} \mathcal{E}_z^{-\nu/2} \overbrace{\mathcal{E}_z^{\nu z} \mathcal{E}_z^{-\nu/2}}^* \mathfrak{I}}_{\mathfrak{I}}$$

$$\mathfrak{L} = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} \underbrace{\mathfrak{L} \mathcal{E}_z^{\nu z} \mathcal{E}_z^{-\nu/2}} \overbrace{\mathcal{E}_z^{\nu z} \mathcal{E}_z^{-\nu/2}}^* = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} z \mathcal{E}_z^{-\nu} \mathfrak{L} \mathcal{E}_z^\nu \mathcal{E}_z^{*\nu}$$

$$\text{tr } \mathfrak{L} = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} \underbrace{\mathcal{E}_z^{\nu z} \mathcal{E}_z^{-\nu/2}} \overline{\mathfrak{L}} \underbrace{\mathcal{E}_z^{\nu z} \mathcal{E}_z^{-\nu/2}} = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} z \mathcal{E}_z^{-\nu} \mathcal{E}_z^\nu \overline{\mathfrak{L}} \mathcal{E}_z^\nu = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} z \mathcal{E}_z^{-\nu} \overbrace{\mathfrak{L} \mathcal{E}_z^\nu}^z$$

$$\text{tr } \mathfrak{s}_x^\nu \overline{y}^{1-\alpha} = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} z \mathcal{E}_z^{-\nu} \mathcal{E}_z^\nu \overline{\mathfrak{s}_x^\nu \overline{y}^{1-\alpha}} \mathcal{E}_z^\nu = \nu^d \int_{dz/\pi^d}^{\mathbb{C}^d} z \mathcal{E}_z^{-\nu} \overbrace{\mathfrak{s}_x^\nu \overline{y}^{1-\alpha}}^z \mathcal{E}_z^\nu$$