

$$\begin{array}{ccccc}
2^n \mathbb{K} & \xrightarrow{\quad \mathbf{x} \quad} & \mathfrak{E}|_{2^n} \mathbb{K} & \xleftarrow{\quad \models \quad} & \mathbb{K}^{2^n} \\
\downarrow \mathfrak{l}^\bullet & & \downarrow Ad \mathfrak{l}^\bullet & & \downarrow \cdot \mathfrak{l} \\
\mathfrak{L} \triangleleft \mathbb{K}^N & \xrightarrow{\quad \mathbf{x} \quad} & \mathfrak{E}|_{\mathfrak{L} \triangleleft \mathbb{K}^N} \mathbb{K} & \xleftarrow{\quad \models \quad} & \mathbb{K}_N^N \overline{\triangleleft} \mathfrak{L}
\end{array}$$

$$\begin{array}{ccccc}
2^n \mathbb{K} & \xrightarrow{\quad \mathbf{x} \quad} & \Psi|_{2^n} \mathbb{K} & \xleftarrow{\quad \models \quad} & \mathbb{K}^{2^n} \\
\downarrow \mathfrak{l}^\bullet & & \downarrow Ad \mathfrak{l}^\bullet & & \downarrow \cdot \mathfrak{l} \\
\mathfrak{L} \triangleleft \mathbb{K}^N & \xrightarrow{\quad \mathbf{x} \quad} & \Psi|_{\mathfrak{L} \triangleleft \mathbb{K}^N} \mathbb{K} & \xleftarrow{\quad \models \quad} & \mathbb{K}_N^N \overline{\triangleleft} \mathfrak{L}
\end{array}$$

$${}_P \mathfrak{l}^\bullet \models \mathfrak{l}^M = \begin{cases} 0 & P \not\models M \\ \mathfrak{l}^{M \cup P} & P \subset M \end{cases}$$

$${}_{M \cup P} \mathfrak{l}^\bullet \underbrace{{}_P \mathfrak{l}^\bullet \models \mathfrak{l}^M}_{= 1} = \begin{bmatrix} {}_P \mathfrak{l}^\bullet \\ \mathfrak{l}^{M \cup P} \end{bmatrix} \mathfrak{l}^M = \underbrace{{}_M \mathfrak{l}^\bullet \mathfrak{l}^M}_{= 1} = {}_{M \cup P} \mathfrak{l}^\bullet \mathfrak{l}^{M \cup P}$$

$$\begin{array}{ccccc}
2^n \mathbb{K} & \xrightarrow{\quad \mathbf{x} \quad} & \mathfrak{E}|_{2^n} \mathbb{K} & \xleftarrow{\quad \models \quad} & \mathbb{K}^{2^n} \\
\downarrow \mathfrak{l}^\bullet & & \downarrow Ad \mathfrak{l}^\bullet & & \downarrow \cdot \mathfrak{l} \\
\mathfrak{L} \triangleleft \mathbb{K}^N & \xrightarrow{\quad \mathbf{x} \quad} & \mathfrak{E}|_{\mathfrak{L} \triangleleft \mathbb{K}^N} \mathbb{K} & \xleftarrow{\quad \models \quad} & \mathbb{K}_N^N \overline{\triangleleft} \mathfrak{L}
\end{array}$$

$$\begin{array}{ccccc}
2^n \mathbb{K} & \xrightarrow{\quad \boxtimes \quad} & \Psi|_{2^n \mathbb{K}} & \xleftarrow{\quad \models \quad} & \mathbb{K}^{2^n} \\
\downarrow \mathcal{L}^\bullet & & \downarrow Ad \mathcal{L}^\bullet & & \downarrow \mathcal{L} \\
\mathcal{L}^{\frac{\mathbb{N}}{\Delta}} \mathbb{K} & \xrightarrow{\quad \boxtimes \quad} & \Psi|_{\mathcal{L}^{\frac{\mathbb{N}}{\Delta}} \mathbb{K}} & \xleftarrow{\quad \models \quad} & \mathbb{K}^{\frac{\mathbb{N}}{\Delta} \mathcal{L}}
\end{array}$$

$$\begin{aligned}
{}_P \mathcal{L}^\cdot \models \mathcal{L}^M &= \begin{cases} 0 & P \not\models M \\ \overline{P > M \sqcup P} (-1) \mathcal{L}^{M \sqcup P} & P \subset M \end{cases} \\
{}_{M \sqcup P} \mathcal{L}^\cdot \underbrace{\mathcal{L}^\cdot \models \mathcal{L}^M}_{=1} &= \left[{}_P \mathcal{L}^\cdot \atop {}_{M \sqcup P} \mathcal{L}^\cdot \right] \mathcal{L}^M = \overline{P > M \sqcup P} \underbrace{\mathcal{L}^\cdot \mathcal{L}^M}_{=1} = \overline{P > M \sqcup P} {}_{M \sqcup P} \mathcal{L}^\cdot \mathcal{L}^{M \sqcup P}
\end{aligned}$$