$$^{n}\mathbb{K}_n = \bigcup_{\underline{m}} {}^{n}_{\mathsf{C}}\mathbb{K}_n \ltimes \prod_j \mathbb{K}^{m_j}$$

$$^{n}\mathbb{K}_n \ni \dot{\triangleleft\!\Gamma}$$

$$\triangleleft\!\Gamma \sim \blacktriangleleft\!\Gamma \Leftrightarrow \bigvee \begin{cases} \rfloor\!\Gamma \in {}^{n}_{\mathsf{C}}\mathbb{K}_n \\ \rfloor\!\Gamma^{-1} \triangleleft\!\Gamma \, \rfloor\!\Gamma = \blacktriangleleft\!\Gamma \end{cases}$$

$$\triangleleft\!\Gamma \in {}^{n}\mathbb{K}_n \Rightarrow \bigvee \dot{\rfloor\!\Gamma} = \bigcup_j \dot{\rfloor\!\Gamma}_j \underset{\text{basic}}{\subset} \Gamma$$

$$\dot{\rfloor\!\Gamma}_j = \left({}^{m}\rfloor\!\Gamma_j \right) m \in m_j \geqslant 1 \, \lambda_j \text{block}$$

$$^{0}\rfloor\!\Gamma_j \, \triangleleft\!\Gamma = \lambda_j \, {}^{0}\rfloor\!\Gamma_j$$

$$\bigwedge_{m \in m_j \llcorner 0} {}^{m}\rfloor\!\Gamma_j \, \triangleleft\!\Gamma = \lambda_j \, {}^{m}\rfloor\!\Gamma_j + {}^{m-1}\rfloor\!\Gamma_j$$

$$\triangleleft\!\Gamma = \rfloor\!\Gamma^{-1} \begin{bmatrix} \lambda_j & 1 & 0 \\ 0 & \lambda_j & 1 \\ 0 & 0 & \lambda_j \end{bmatrix}$$

$$\frac{^{0}\rfloor\!\Gamma_j}{\lambda_j = \lambda} \underset{\text{basic}}{\subset} \ker \left(\triangleleft\!\Gamma - \lambda 1 \right)$$

$$\frac{^{0}\rfloor\!\Gamma_j}{\lambda_j = 0} \underset{\text{basic}}{\subset} \ker \triangleleft\!\Gamma$$

Case 1 / $\Gamma \neq \operatorname{ran} \surd$

$$\llcorner \xrightarrow{\surd} \operatorname{ran} \surd \underset{\text{ind dim } \Gamma}{\Longrightarrow} \bigvee \dot{\rrbracket} = \bigcup_j \dot{\rrbracket}_j \underset{\text{basic}}{\subset} \operatorname{ran} \surd$$

$$\Longrightarrow \frac{{}^0\rrbracket_j}{\lambda_j = 0} \underset{\text{basic}}{\subset} \ker \surd \cap \operatorname{ran} \surd \Longrightarrow \bigvee_{\lambda_j \text{ blick}} \dot{\underset{\sim}{\rrbracket}} \cup {}^0\rrbracket_j \, \lambda_j = 0 \underset{\text{basic}}{\subset} \ker \surd \bigwedge_j^{\lambda_j = 0} \bigvee {}^{m_j}\rrbracket_j \surd = {}^{m_j - 1}\rrbracket_j$$

$$\bigcup_j^{\lambda_j \neq 0} \dot{\rrbracket}_j \cup \bigcup_j^{\lambda_j = 0} \left(\dot{\rrbracket}_j \cup {}^{m_j}\rrbracket_j \right) \cup \dot{\underset{\sim}{\rrbracket}} \underset{\text{basic}}{\subset} \Gamma$$

$$\text{free} \sum_j \sum_m^{m_j} a_m^j \, {}^m\rrbracket_j = \sum_j^{\lambda_j = 0} b_j \, {}^{m_j}\rrbracket_j + \sum_n c_n \, {}^n\underset{\sim}{\rrbracket} \underset{\text{apply } \surd}{\Longrightarrow} \sum_j \sum_m^{m_j \llcorner 0} a_m^j \left(\lambda_j \, {}^m\rrbracket_j + {}^{m-1}\rrbracket_j \right) + \sum_j^{\lambda_j \neq 0} a_0^j \, {}^0\rrbracket_j$$

$$= \sum_j^{\lambda_j = 0} b_j \, {}^{m_j - 1}\rrbracket_j$$

$$\Longrightarrow \bigwedge_j b_j = 0 \Longrightarrow \sum_j \sum_j^{m_j} a_m^j \, {}^m\rrbracket_j = \sum_n c_n \, {}^n\underset{\sim}{\rrbracket} \in \operatorname{ran} \surd \cap \ker \surd \Longrightarrow \begin{cases} \bigwedge_m^{m_j \llcorner 0} a_m^j = 0 \\ \bigwedge_j^{\lambda_j \neq 0} a_0^j = 0 \end{cases}$$

$$\Longrightarrow \sum_j^{\lambda_j = 0} a_0^j \, {}^0\rrbracket_j = \sum_n c_n \, {}^n\underset{\sim}{\rrbracket} \Longrightarrow \bigwedge_j^{\lambda_j = 0} a_0^j = 0$$

$$\bigwedge_n c_n = 0 \Longrightarrow \text{free} \Longrightarrow \text{distinct}$$

$$\Longrightarrow \sharp \dot{\rrbracket} + \sharp \frac{{}^0\rrbracket_j}{\lambda_j = 0} + \sharp \dot{\underset{\sim}{\rrbracket}} = \dim \operatorname{ran} \surd + \dim \ker \surd = \dim \surd \Longrightarrow \text{full}$$

$$\ker \surd \underset{\text{basic}}{\supset} \frac{{}^0\rrbracket_j}{\lambda_j = 0} \cup \dot{\underset{\sim}{\rrbracket}}$$

$$\bigwedge_{\lambda \neq 0} \operatorname{ran} \surd \supset \ker (\surd - \lambda 1) \underset{\text{basic}}{\supset} \frac{{}^0\rrbracket_j}{\lambda_j = \lambda}$$

Case 2 / $\Gamma = \operatorname{ran} \surd \Longrightarrow \bigvee_{\lambda \in \mathbb{K}} \ker (\surd - \lambda 1) \neq 0 \Longrightarrow \surd - \lambda 1 = S^{-1} J S \Longrightarrow \surd = S^{-1} (J + \lambda 1) S$