$$\mathtt{J} \in {}^n\mathbb{R}_n \text{ self-adj } \Rightarrow \begin{cases} \bigvee\limits_{\text{ONB}} \mathbb{J} = \begin{bmatrix} {}^1\mathbb{J} \\ + \\ {}^n\mathbb{J} \end{bmatrix} \in {}^n\mathbb{K}_n \\ {}^j\mathbb{J}\,\mathtt{J} = \lambda_j\, {}^j\mathbb{J} \text{ eig-vect} \end{cases} \Rightarrow \mathtt{J} = \mathbb{J}^* \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \dagger & 0 \\ 0 & 0 & \lambda_n \end{bmatrix} \mathbb{J}$$

$$\mathbb{K} = \mathbb{R}: \quad \mathtt{J} = \mathtt{J}^*$$

$$\text{energy } {}^{\mathsf{P}}\mathcal{E} = \frac{\mathsf{P}\,\mathtt{J} \ast \mathsf{P}}{\mathsf{P} \ast \mathsf{P}} \Rightarrow \mathbb{R}_n \llcorner 0 \xrightarrow[\text{stet}]{\mathcal{E}} \mathbb{R}$$

$$\Gamma\, {}^{\mathsf{P}}\underline{\mathcal{E}} = \frac{(\Gamma\,\mathtt{J} \ast \mathsf{P} + \mathsf{P}\,\mathtt{J} \ast \Gamma)\, \mathsf{P} \ast \mathsf{P} - \mathsf{P}\,\mathtt{J} \ast \mathsf{P}\, (\Gamma \ast \mathsf{P} + \mathsf{P} \ast \Gamma)}{(\mathsf{P} \ast \mathsf{P})^2} = 2 \frac{(\mathsf{P}\,\mathtt{J} \ast \Gamma)(\mathsf{P} \ast \mathsf{P}) - (\mathsf{P}\,\mathtt{J} \ast \mathsf{P})(\mathsf{P} \ast \Gamma)}{(\mathsf{P} \ast \mathsf{P})^2}$$

$$\text{bes abg } \mathbb{S}_{n-1} = \frac{\mathsf{P} \in \mathbb{R}_n}{\mathsf{P} \ast \mathsf{P} = 1} \text{ cpt } \Rightarrow \bigvee_{{}^1\mathbb{J} \in \mathbb{S}_{n-1}} \bigwedge_{\mathsf{P} \in \mathbb{S}_{n-1}} {}^{{}^1\mathbb{J}}\mathcal{E} \leqslant {}^{\mathsf{P}}\mathcal{E} \Rightarrow \bigwedge_{\mathsf{P} \in \mathbb{R}_n \llcorner 0} {}^{\mathsf{P}}\mathcal{E} = {}^{\mathsf{P}/(\mathsf{P} \ast \mathsf{P})^{1/2}}\mathcal{E} \geqslant {}^{{}^1\mathbb{J}}\mathcal{E}$$

$$\Rightarrow {}^{{}^1\mathbb{J}}\mathcal{E} \leqslant {}^{{}^1\mathbb{J} + t\mathsf{P}}\mathcal{E} \text{ um 0 min at } t = 0 \Rightarrow 0 = {}^0_t\partial\, {}^{{}^1\mathbb{J} + t\Gamma}\mathcal{E} = {}^{{}^1\mathbb{J}}_{\Gamma}\underline{\mathcal{E}} = 2 \frac{\underbrace{{}^1\mathbb{J}\,\mathtt{J} \ast \Gamma}\,\underbrace{{}^1\mathbb{J} \ast {}^1\mathbb{J}} - \underbrace{{}^1\mathbb{J}\,\mathtt{J} \ast {}^1\mathbb{J}}\,\underbrace{{}^1\mathbb{J} \ast \Gamma}}{\overbrace{{}^1\mathbb{J} \ast {}^1\mathbb{J}}^2}$$

$$= 2\, \overbrace{{}^1\mathbb{J}\,\mathtt{J} \ast \Gamma - \underbrace{{}^1\mathbb{J}\,\mathtt{J} \ast {}^1\mathbb{J}}\,\underbrace{{}^1\mathbb{J} \ast \Gamma}} = 2\, \overbrace{{}^1\mathbb{J}\,\mathtt{J} - \underbrace{{}^1\mathbb{J}\,\mathtt{J} \ast {}^1\mathbb{J}}\, {}^1\mathbb{J}} \ast \Gamma \underset{\Gamma \text{ bel}}{\Rightarrow} {}^1\mathbb{J}\,\mathtt{J} = \underbrace{{}^1\mathbb{J}\,\mathtt{J} \ast {}^1\mathbb{J}}\, {}^1\mathbb{J} = \lambda_1\, {}^1\mathbb{J}$$

$$\mathbb{K} = \mathbb{C}: \quad \det(\mathtt{J} - \lambda I) \text{ has zero } \lambda_1 \Rightarrow \begin{cases} \bigvee {}^1\mathbb{J} \in \mathbb{C}_n \\ {}^1\mathbb{J} \ast {}^1\mathbb{J} = 1 \end{cases} \quad {}^1\mathbb{J}\,\mathtt{J} = \lambda_1\, {}^1\mathbb{J}$$

$$\mathbb{K}_n \sqsupset {}^1\mathbb{J}^{\perp} \xrightarrow{\mathtt{J}} {}^1\mathbb{J}^{\perp} \underset{\text{ind}}{\Rightarrow} \begin{cases} \bigvee\limits_{\text{ONB}} {}^2\mathbb{J} \cdots {}^n\mathbb{J} \in {}^1\mathbb{J}^{\perp} \\ {}^j\mathbb{J}\,\mathtt{J} = \lambda_j\, {}^j\mathbb{J} \end{cases} \Rightarrow \begin{cases} \text{ONB } {}^1\mathbb{J} \cdots {}^n\mathbb{J} \in \mathbb{K}_n \\ \mathtt{J}\, {}^j\mathbb{J} = \lambda_j\, {}^j\mathbb{J} \end{cases}$$

$$\mathbb{J}\,\mathtt{J} = \begin{bmatrix} {}^1\mathbb{J}\,\mathtt{J} \\ \cdot \\ {}^n\mathbb{J}\,\mathtt{J} \end{bmatrix} = \begin{bmatrix} \lambda_1\, {}^1\mathbb{J} \\ \cdot \\ \lambda_n\, {}^n\mathbb{J} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \cdot & 0 \\ 0 & 0 & \lambda_n \end{bmatrix} \mathbb{J}$$

$$ʃ = \overset{*}{ʃ} \ \text{herm}$$

$$Γ ⊼ ↾ = (↾ ⊼ Γ)^*$$

$$\Rightarrow \bigvee {}^1ʃ \cdots {}^nʃ \underset{\text{basic}}{\subset} \mathbb{K}_n \, ʃ = ʃ \begin{array}{ccc|c} \lambda_1 & 0 & 0 & \\ 0 & \dagger & 0 & 0 \\ 0 & 0 & \lambda_r & \\ \hline & 0 & & 0 \end{array} ʃ^*$$

$$\lambda_i \neq 0$$

$$\Gamma = \Gamma^{\sharp} \text{ symm}$$

$$\Gamma \ast \Gamma = \Gamma \ast \Gamma$$

$$\Rightarrow \bigvee {}^1\Gamma \cdots {}^n\Gamma \underset{\text{basic}}{\subset} \mathbb{K}_n \; \Gamma = \Gamma \begin{array}{ccc|c} \lambda_1 & 0 & 0 & \\ 0 & \dagger & 0 & 0 \\ 0 & 0 & \lambda_r & \\ \hline & 0 & & 0 \end{array} \Gamma^{\sharp}$$

$$\lambda_i \neq 0$$

$$\max \text{ free } {}^1\Gamma \cdots {}^k\Gamma \in \mathbb{K}_n$$

$${}^i\Gamma \ast {}^j\Gamma = {}^i\delta_j \, \lambda_i$$

$$\lambda_i \neq 0$$

$$\Gamma_0 =< {}^1\Gamma \cdots {}^k\Gamma > \subset \mathbb{K}_n$$

$$\dim \Gamma_0 = k$$

$$\Gamma_0 \cap \Gamma_0^{\perp} = \ker \ast_{\Gamma_0} = \ker \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \dagger & 0 \\ 0 & 0 & \lambda_k \end{bmatrix} = 0$$

$$\Gamma \in \Gamma \Rightarrow \Gamma := \Gamma - \sum_i {}^i\Gamma \, \lambda_i^{-1} \Gamma \ast {}^i\Gamma \Rightarrow \Gamma \ast {}^j\Gamma = \Gamma \ast {}^j\Gamma - \sum_i {}^i\Gamma \ast {}^j\Gamma \, \lambda_i^{-1} \Gamma \ast {}^i\Gamma$$

$$= \Gamma \ast {}^j\Gamma - {}^j\Gamma \ast {}^j\Gamma \, \lambda_j^{-1} \Gamma \ast {}^j\Gamma = 0 \Rightarrow \Gamma \in \Gamma_0^{\perp} \Rightarrow \Gamma = \Gamma_0 \times \Gamma_0^{\perp}$$

$$\text{any basis } {}^{k+1}\Gamma \cdots {}^n\Gamma \in \Gamma_0^{\perp}$$

$$\lightning \; \Gamma_0^{\perp} \ast \Gamma_0^{\perp} \neq 0 \Rightarrow \bigvee \dot{\Gamma} \in \Gamma_0^{\perp}$$

$$0 \neq \Gamma \ast \Gamma = \frac{1}{2} \overbrace{\underbrace{\Gamma + \Gamma} \ast \underbrace{\Gamma + \Gamma} - \Gamma \ast \Gamma - \Gamma \ast \Gamma}$$

$$\Rightarrow \bigvee {}^{k+1}\Gamma \in \Gamma_0^{\perp} \; {}^{k+1}\Gamma \ast {}^{k+1}\Gamma \neq 0 \Rightarrow {}^1\Gamma \cdots {}^{k+1}\Gamma \text{ orthog } \lightning \Rightarrow \Gamma_0^{\perp} \ast \Gamma_0^{\perp} = 0$$

$$\urcorner^{ij} = \underbrace{\llcorner \llcorner^{\sharp}} \delta_{ij} - \llcorner^i \llcorner^j$$

$$\text{ʅ} = \begin{array}{c|c|c} 5 & 0 & 6 \\ \hline 0 & 5/2 & 0 \\ \hline 6 & 0 & 15 \end{array}$$

$$\det(\text{ʅ} - \lambda I) = \det \begin{array}{c|c|c} 5-\lambda & 0 & 6 \\ \hline 0 & 5/2-\lambda & 0 \\ \hline 6 & 0 & 15-\lambda \end{array} = \overbrace{5/2-\lambda}\,\overbrace{\underbrace{5-\lambda}\,\underbrace{15-\lambda} - 36}$$

$$= \overbrace{5/2-\lambda}\,\overbrace{\lambda - \underbrace{10+\sqrt{61}}}\,\overbrace{\lambda - \underbrace{10-\sqrt{61}}}$$

$$\begin{array}{c|c|c} 5 & 0 & 6 \\ \hline 0 & 5/2 & 0 \\ \hline 6 & 0 & 15 \end{array} \begin{bmatrix} \underbrace{\sqrt{61} - 5\varkappa} z \\ 0 \\ 6\varkappa z \end{bmatrix} = \underbrace{10 + \varkappa\sqrt{61}} \begin{bmatrix} \underbrace{\sqrt{61} - 5\varkappa} z \\ 0 \\ 6\varkappa z \end{bmatrix}$$

$$\begin{array}{c|c|c} 5 & 0 & 6 \\ \hline 0 & 5/2 & 0 \\ \hline 6 & 0 & 15 \end{array} \begin{bmatrix} 0 \\ y \\ 0 \end{bmatrix} = \frac{5}{2} \begin{bmatrix} 0 \\ y \\ 0 \end{bmatrix}$$

$$\text{ʅ} = \begin{array}{c|c|c} \underbrace{\sqrt{61} - 5} z & 0 & \underbrace{\sqrt{61} + 5} z \\ \hline 0 & y & 0 \\ \hline 6z & 0 & -6z \end{array}$$

$$\overset{\sharp}{\text{ʅ}}\ \text{ʅ}\ \text{ʅ} = \begin{array}{c|c|c} 10+\sqrt{61} & 0 & 0 \\ \hline 0 & 5/2 & 0 \\ \hline 0 & 0 & 10-\sqrt{61} \end{array}$$