

$$\text{graded VR } \mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} = \frac{\mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} \mid \mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot -}}{\mathbb{L}^{-} \triangleleft \mathbb{L}^{\cdot} \mid \mathbb{L}^{-} \triangleleft \mathbb{L}^{\cdot -}} = \frac{\frac{\mathbb{L} \mid \mathbb{L}}{\mathbb{L} \mid \mathbb{L}}}{\frac{\mathbb{L} \in \mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} \mid \mathbb{L} \in \mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot -}}{\mathbb{L} \in \mathbb{L}^{-} \triangleleft \mathbb{L}^{\cdot} \mid \mathbb{L} \in \mathbb{L}^{-} \triangleleft \mathbb{L}^{\cdot -}}}$$

$$\mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} = \left\{ \frac{\mathbb{L} \mid 0}{0 \mid \mathbb{L}} \right\} = \frac{\mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} \mid 0}{0 \mid \mathbb{L}^{-} \triangleleft \mathbb{L}^{\cdot -}}$$

$$\mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} = \left\{ \frac{0 \mid \mathbb{L}}{\mathbb{L} \mid 0} \right\} = \frac{0 \mid \mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot -}}{\mathbb{L}^{-} \triangleleft \mathbb{L}^{\cdot} \mid 0}$$

$$\mathbb{L}^{\cdot} \otimes \mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} \xrightarrow{\text{even}} \mathbb{L}^{\cdot}$$

$$\mathbb{L} \mid \mathbb{L} \otimes \frac{\mathbb{L} \mid \mathbb{L}}{\mathbb{L} \mid \mathbb{L}} \not\sim \mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L} \mid \mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L}$$

$$\underbrace{\mathbb{L}^{\cdot} \triangleleft \mathbb{K} \otimes \mathbb{L}^{\cdot}}_{\cong} \xrightarrow{\cong} \mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot}$$

$$\underbrace{\mathbb{L}^{\cdot} \times \mathbb{L}^{-} \triangleleft \mathbb{K} \otimes \mathbb{L}^{\cdot} \times \mathbb{L}^{-}}_{\cong} = \underbrace{\mathbb{L}^{\cdot} \triangleleft \mathbb{K} \otimes \mathbb{L}^{\cdot}}_{\cong \mathbb{L}} \times \underbrace{\mathbb{L}^{-} \triangleleft \mathbb{K} \otimes \mathbb{L}^{-}}_{\cong \mathbb{L}^{-}} \times \underbrace{\mathbb{L}^{\cdot} \triangleleft \mathbb{K} \otimes \mathbb{L}^{-}}_{\cong \mathbb{L}^{-}} \times \underbrace{\mathbb{L}^{-} \triangleleft \mathbb{K} \otimes \mathbb{L}^{\cdot}}_{\cong \mathbb{L}}$$

$$\frac{\mathbb{L} \mid \mathbb{L}}{\mathbb{L} \mid \mathbb{L}} \frac{\mathbb{L} \mid \mathbb{L}}{\mathbb{L} \mid \mathbb{L}} = \frac{\mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L} \mid \mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L}}{\mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L} \mid \mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L}}$$