

$$\mathcal{L} = - \mathcal{L}^t \in {}_2^n\mathbb{B}_n^{\Omega}$$

$$\text{Pf } \mathcal{L} = \frac{1}{n!2^n} \sum_{\pi \in \mathcal{S}_n} {}^1\pi \mathcal{L}_{2^\pi} \cdots {}^{n-1}\pi \mathcal{L}_{n^\pi}$$

$$\text{Pf } \begin{array}{cc|cc} 0 & a & b & c \\ -a & 0 & d & e \\ \hline -b & -d & 0 & f \\ -c & -e & -f & 0 \end{array} = af - be + cd$$

$$\det \begin{array}{cc|cc} 0 & a & b & c \\ -a & 0 & d & e \\ \hline -b & -d & 0 & f \\ -c & -e & -f & 0 \end{array} = a \begin{bmatrix} a & b & c \\ -d & 0 & f \\ -e & -f & 0 \end{bmatrix} - b \begin{bmatrix} a & b & c \\ 0 & d & e \\ -e & -f & 0 \end{bmatrix} + c \begin{bmatrix} a & b & c \\ 0 & d & e \\ -d & 0 & f \end{bmatrix}$$

$$= a(-bfe + cdf + adf) - b(-bee + cde + aef) + c(adf - bed + cdd) = (af - be + cd)^2$$