

$$\mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} = \frac{\mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} \mid \mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{-}}{\mathbb{L}^{-} \triangleleft \mathbb{L}^{\cdot} \mid \mathbb{L}^{-} \triangleleft \mathbb{L}^{-}} \frac{\mathbb{L} \mid \mathbb{L}}{\mathbb{L} \mid \mathbb{L}}$$

$$\mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} = \left\{ \frac{\mathbb{L} \mid 0}{0 \mid \mathbb{L}} \right\} = \frac{\mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} \mid 0}{0 \mid \mathbb{L}^{-} \triangleleft \mathbb{L}^{-}}$$

$$\mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} = \left\{ \frac{0 \mid \mathbb{L}}{\mathbb{L} \mid 0} \right\} = \frac{0 \mid \mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{-}}{\mathbb{L}^{-} \triangleleft \mathbb{L}^{\cdot} \mid 0}$$

$$\mathbb{L}^{\cdot} \otimes \mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot} \xrightarrow{\text{even}} \mathbb{L}^{\cdot}$$

$$\mathbb{L} \mid \mathbb{L} \otimes \frac{\mathbb{L} \mid \mathbb{L}}{\mathbb{L} \mid \mathbb{L}} \neq \mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L} \mid \mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L}$$

$$\underbrace{\mathbb{L}^{\cdot} \triangleleft \mathbb{K} \otimes \mathbb{L}^{\cdot}}_{\cong} \xrightarrow{\cong} \mathbb{L}^{\cdot} \triangleleft \mathbb{L}^{\cdot}$$

$$\underbrace{\mathbb{L}^{\cdot} \times \mathbb{L}^{-}}_{\triangleleft \mathbb{K} \otimes \mathbb{L}^{\cdot} \times \mathbb{L}^{\cdot} \times \mathbb{L}^{-}} = \underbrace{\mathbb{L}^{\cdot} \triangleleft \mathbb{K} \otimes \mathbb{L}^{\cdot}}_{\cong \mathbb{L}} \times \underbrace{\mathbb{L}^{-} \triangleleft \mathbb{K} \otimes \mathbb{L}^{-}}_{\cong \mathbb{L}^{-}} \times \underbrace{\mathbb{L}^{\cdot} \triangleleft \mathbb{K} \otimes \mathbb{L}^{-}}_{\cong \mathbb{L}^{-}} \times \underbrace{\mathbb{L}^{-} \triangleleft \mathbb{K} \otimes \mathbb{L}^{\cdot}}_{\cong \mathbb{L}}$$

$$\frac{\mathbb{L} \mid \mathbb{L} \quad \mathbb{L} \mid \mathbb{L} \quad \mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L} \mid \quad \mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L}}{\mathbb{L} \mid \mathbb{L} \quad \mathbb{L} \mid \mathbb{L} \quad \mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L} \mid \quad \mathbb{L} \mathbb{L} + \mathbb{L} \mathbb{L}}$$

$$\frac{\mathbb{L} \mid 0 \quad \mathbb{L} \mid 0}{0 \mid \mathbb{L} \quad 0 \mid \mathbb{L}} = \frac{\mathbb{L} \mathbb{L} \mid 0}{0 \mid \mathbb{L} \mathbb{L}} \quad \text{ev}$$

$$\frac{\mathbb{L} \mid 0 \quad 0 \mid \mathbb{L}}{0 \mid \mathbb{L} \quad \mathbb{L} \mid 0} = \frac{0 \mid \mathbb{L} \mathbb{L}}{\mathbb{L} \mathbb{L} \mid 0} \quad \text{odd}$$

$$\frac{0 \mid \mathbb{L} \quad \mathbb{L} \mid 0}{\mathbb{L} \mid 0 \quad 0 \mid \mathbb{L}} = \frac{0 \mid \mathbb{L} \mathbb{L}}{\mathbb{L} \mathbb{L} \mid 0} \quad \text{odd}$$

$$\frac{0 \mid \mathbb{L} \quad 0 \mid \mathbb{L}}{\mathbb{L} \mid 0 \quad \mathbb{L} \mid 0} = \frac{\mathbb{L} \mathbb{L} \mid 0}{0 \mid \mathbb{L} \mathbb{L}} \quad \text{ev}$$

graded commutator

$$\begin{aligned}
\frac{\begin{array}{c|c} \mathbb{L} & \mathbb{V} \\ \hline \mathbb{1} & \mathbb{1} \end{array}} * \frac{\begin{array}{c|c} \mathbb{E} & \mathbb{V} \\ \hline \mathbb{4} & \mathbb{4} \end{array}} &= \frac{\begin{array}{c|c} \mathbb{L} & \mathbb{V} \\ \hline \mathbb{1} & \mathbb{1} \end{array}} * \frac{\begin{array}{c|c} \mathbb{E} & \mathbb{V} \\ \hline \mathbb{4} & \mathbb{4} \end{array}} - {}^{ij} \frac{\begin{array}{c|c} \mathbb{E} & \mathbb{V} \\ \hline \mathbb{4} & \mathbb{4} \end{array}} \frac{\begin{array}{c|c} \mathbb{L} & \mathbb{V} \\ \hline \mathbb{1} & \mathbb{1} \end{array}} \\
&= \frac{\begin{array}{c|c} \mathbb{L} \mathbb{E} + \mathbb{V} \mathbb{4} - \mathbb{E} \mathbb{L} + \mathbb{V} \mathbb{1} & \mathbb{V} \mathbb{4} + \mathbb{L} \mathbb{V} - \mathbb{E} \mathbb{1} - \mathbb{E} \mathbb{V} \\ \hline \mathbb{1} \mathbb{E} + \mathbb{1} \mathbb{4} - \mathbb{4} \mathbb{L} - \mathbb{4} \mathbb{1} & \mathbb{1} \mathbb{4} + \mathbb{1} \mathbb{V} + \mathbb{4} \mathbb{1} - \mathbb{4} \mathbb{V} \end{array}} \\
&\quad \text{graded Lie alg}
\end{aligned}$$