$$w = u + iv$$

$$\exp\left(z_w \overset{*}{\alpha} - \zeta_w \overset{*}{\zeta}_w / 2\right)$$

$$^{\zeta}\text{ᛏ} = {}^{\zeta}\text{ɿ}\ {}^{-\zeta\overset{*}{\zeta}/2}\mathfrak{e}$$

$$^{\zeta}_{e}E_{\vartheta} = {}^{\zeta\overset{*}{\vartheta} - \vartheta\overset{*}{\vartheta}/2 - \zeta\overset{*}{\zeta}/2}\mathfrak{e} : \quad {}^{z}\text{ᛏ} = \int_{d\vartheta}^{\mathbb{C}^n} {}^{\zeta\overset{*}{\vartheta} - \vartheta\overset{*}{\vartheta}/2 - \zeta\overset{*}{\zeta}/2}\mathfrak{e}\ {}^{\vartheta}\text{ᛏ}$$

$$^{\zeta}\text{ɿ} = \int_{d\vartheta}^{\mathbb{C}^n} {}^{-\vartheta\overset{*}{\vartheta}}\mathfrak{e}\ {}^{\zeta\overset{*}{\vartheta}}\mathfrak{e}\ {}^{\vartheta}\text{ɿ} = \int_{d\vartheta}^{\mathbb{C}^n} {}^{(\zeta - \vartheta)\overset{*}{\vartheta}}\mathfrak{e}\ {}^{\vartheta}\text{ɿ}$$

$$\Longrightarrow \text{ LHS } = \int_{d\vartheta}^{\mathbb{C}^n} {}^{(\zeta - \vartheta)\overset{*}{\vartheta}}\mathfrak{e}\ {}^{\vartheta}\text{ɿ}\ {}^{-\zeta\overset{*}{\zeta}/2}\mathfrak{e} = \int_{d\vartheta}^{\mathbb{C}^n} {}^{(\zeta - \vartheta)\overset{*}{\vartheta}}\mathfrak{e}\ {}^{\vartheta}\text{ᛏ}\ {}^{\vartheta\overset{*}{\vartheta}/2 - \zeta\overset{*}{\zeta}/2}\mathfrak{e} = \text{ RHS}$$

$$^{\zeta}_{w}E_{\vartheta} = {}^{\zeta_w}_{e}E_{\vartheta_w} = \exp\left(\zeta_w \overset{*}{\vartheta}_w - \vartheta_w \overset{*}{\vartheta}_w / 2 - \zeta_w \overset{*}{\zeta}_w / 2\right)$$

$$\alpha_w \overset{*}{\beta}_w = \overbrace{\alpha + \bar{\alpha} + \underbrace{\alpha - \bar{\alpha}} w}\ \bar{u}^{1}\ \overbrace{\overset{*}{\beta} + \overset{t}{\beta} + \overset{*}{w} \underbrace{\overset{*}{\beta} - \overset{t}{\beta}}}$$

$$\text{LHS } = \overbrace{\alpha \underbrace{1 + w} + \bar{\alpha} \underbrace{1 - w}}\ {}^{-1/2}\bar{u}\ {}^{-1/2}\bar{u}\ \overset{*}{\overbrace{\beta \underbrace{1 + w} + \bar{\beta} \underbrace{1 - w}}} = \overbrace{\alpha \underbrace{1 + w} + \bar{\alpha} \underbrace{1 - w}}\ \bar{u}^{1}\ \overbrace{\underbrace{1 + \overset{*}{w}} \overset{*}{\beta} + \underbrace{1 - \overset{*}{w}} \overset{t}{\beta}} = \text{ RHS}$$

$$\begin{bmatrix} \zeta_g & \bar{\zeta}_g \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \zeta & \bar{\zeta} \end{bmatrix} \begin{array}{c|c} \overbrace{a + d} + i \overbrace{b - c} & \overbrace{a - d} - i \overbrace{b + c} \\ \hline \underbrace{a - d} + i \underbrace{b + c} & \underbrace{a + d} + i \underbrace{c - b} \end{array}$$

$$\begin{bmatrix} \xi_g & \eta_g \end{bmatrix} = \begin{bmatrix} \xi & \eta \end{bmatrix} \begin{array}{c|c} a & b \\ \hline c & d \end{array}$$

$$\Longrightarrow \text{ LHS } = \begin{bmatrix} \xi_g & \eta_g \end{bmatrix} \begin{array}{c|c} 1 & 1 \\ \hline i & -i \end{array} = \begin{bmatrix} \xi & \eta \end{bmatrix} \begin{array}{c|c} a & b \\ \hline c & d \end{array} \begin{array}{c|c} 1 & 1 \\ \hline i & -i \end{array} = \frac{1}{2} \begin{bmatrix} \zeta & \bar{\zeta} \end{bmatrix} \begin{array}{c|c} 1 & -i \\ \hline 1 & i \end{array} \begin{array}{c|c} a & b \\ \hline c & d \end{array} \begin{array}{c|c} 1 & 1 \\ \hline i & -i \end{array} = \text{ RHS}$$

$$\zeta_g = \frac{\zeta + \bar{\zeta}}{2} \underbrace{a + ib} + \frac{\zeta - \bar{\zeta}}{2} \underbrace{d - ic}$$

$$\left[\begin{array}{c|c} 1 & 0 \\ \hline -v & u \end{array}\right]\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline 0 & u^{-1/2} \end{array}\right] = \left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline -v\,u^{-1/2} & u^{1/2} \end{array}\right] \in {}^{r}_{2}\mathbb{R}^{\Omega}_{r}$$

$$\left[\begin{array}{c|c} u^{-1/2} & -u^{-1/2}v \\ \hline 0 & u^{1/2} \end{array}\right]\left[\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}\right]\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline -v\,u^{-1/2} & u^{1/2} \end{array}\right] = \left[\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}\right]$$

$$\begin{bmatrix}\xi_w & \eta_w\end{bmatrix} = \begin{bmatrix}\xi & \eta\end{bmatrix}\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline -v\,u^{-1/2} & u^{1/2} \end{array}\right]$$

$$\frac{1}{2}\left[\begin{array}{c|c} 1+w & 1-\bar{w} \\ \hline 1-w & 1+\bar{w} \end{array}\right]\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline 0 & u^{-1/2} \end{array}\right] \in {}^{r}_{2}\mathbb{C}^{\Omega}_{r}$$

$$\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline 0 & u^{-1/2} \end{array}\right]\left[\begin{array}{c|c} 1+w & 1-w \\ \hline 1-\bar{w} & 1+\bar{w} \end{array}\right]\left[\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}\right]\left[\begin{array}{c|c} 1+w & 1-\bar{w} \\ \hline 1-w & 1+\bar{w} \end{array}\right]\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline 0 & u^{-1/2} \end{array}\right]$$

$$= 2\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline 0 & u^{-1/2} \end{array}\right]\left[\begin{array}{c|c} 0 & w+\bar{w} \\ \hline -w-\bar{w} & 0 \end{array}\right]\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline 0 & u^{-1/2} \end{array}\right] = 4\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline 0 & u^{-1/2} \end{array}\right]\left[\begin{array}{c|c} 0 & u \\ \hline -u & 0 \end{array}\right]\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline 0 & u^{-1/2} \end{array}\right] = 4\left[\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}\right]$$

$$2\begin{bmatrix}\zeta_w & \bar{\zeta}_w\end{bmatrix} = \begin{bmatrix}\zeta & \bar{\zeta}\end{bmatrix}\left[\begin{array}{c|c} 1+w & 1-\bar{w} \\ \hline 1-w & 1+\bar{w} \end{array}\right]\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline 0 & u^{-1/2} \end{array}\right]$$

$$\begin{bmatrix}\zeta_w & \bar{\zeta}_w\end{bmatrix} = \begin{bmatrix}\xi_w & \eta_w\end{bmatrix}\left[\begin{array}{c|c} 1 & 1 \\ \hline i & -i \end{array}\right]$$

$$2\begin{bmatrix}\xi & \eta\end{bmatrix} = \begin{bmatrix}\zeta & \bar{\zeta}\end{bmatrix}\left[\begin{array}{c|c} 1 & -i \\ \hline 1 & i \end{array}\right]$$

$$\text{LHS} = \begin{bmatrix}\xi_w & \eta_w\end{bmatrix}\left[\begin{array}{c|c} 1 & 1 \\ \hline i & -i \end{array}\right] = \begin{bmatrix}\xi & \eta\end{bmatrix}\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline -v\,u^{-1/2} & u^{1/2} \end{array}\right]\left[\begin{array}{c|c} 1 & 1 \\ \hline i & -i \end{array}\right] = \begin{bmatrix}\zeta & \bar{\zeta}\end{bmatrix}\left[\begin{array}{c|c} 1 & -i \\ \hline 1 & i \end{array}\right]\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline -v\,u^{-1/2} & u^{1/2} \end{array}\right]\left[\begin{array}{c|c} 1 & 1 \\ \hline i & -i \end{array}\right]$$

$$= \begin{bmatrix}\zeta & \bar{\zeta}\end{bmatrix}\left[\begin{array}{c|c} u^{-1/2} + u^{1/2} + iv\,u^{-1/2} & u^{-1/2} - u^{1/2} + iv\,u^{-1/2} \\ \hline u^{-1/2} - u^{1/2} - iv\,u^{-1/2} & u^{-1/2} + u^{1/2} - iv\,u^{-1/2} \end{array}\right] = \begin{bmatrix}\zeta & \bar{\zeta}\end{bmatrix}\left[\begin{array}{c|c} 1+u+iv & 1-u+iv \\ \hline 1-u-iv & 1+u-iv \end{array}\right]\left[\begin{array}{c|c} u^{-1/2} & 0 \\ \hline 0 & u^{-1/2} \end{array}\right] = \text{RHS}$$

$$J_w = \left[\begin{array}{c|c} v\,u^{-1} & -u - v\,u^{-1}v \\ \hline u^{-1} & -u^{-1}v \end{array}\right]$$

$$
\begin{array}{ccc}
\mathbb{C}_{2n} & \xrightarrow{\;u^{-1/2}\;} & \mathbb{C}_n \\
\Big\downarrow{\scriptstyle J_w} & & \Big\downarrow{\scriptstyle i} \\
\mathbb{C}_{2n} & \xrightarrow[\;u^{-1/2}\;]{} & \mathbb{C}_n
\end{array}
$$