

$$\begin{aligned} & \mathbb{K} = \mathbb{R}:\mathbb{C}:\mathbb{H} \\ & {}^m\mathbb{K}_n \supset \overbrace{{}^m\mathbb{K}_m \times {}^n\mathbb{K}_n}^{\text{sing val}} \xrightarrow{\simeq} \tilde{\mathbb{R}}_+^{m \wedge n} \\ & {}^m\mathbb{K}_n \xleftarrow{\simeq} {}^m\mathbb{K}_m \times \underbrace{\tilde{\mathbb{R}}_+^{m \wedge n}} \times {}^n\mathbb{K}_n \end{aligned}$$

$$\mathbb{F}_0 \ni \mathbb{F} \ni \mathbb{F} \in {}^m\mathbb{K}_n$$

$$\mathbb{F} \sim \mathbb{F} \Leftrightarrow \bigvee \left\{ \begin{array}{l} \mathbb{F} \in {}^m\mathbb{K}_m \\ \mathbb{F} \in {}^n\mathbb{K}_n \end{array} \right. \quad \mathbb{F}^* \mathbb{F} \mathbb{F} = \mathbb{F}$$

$$\mathbb{F} \in {}^m\mathbb{K}_n \Rightarrow \bigvee \left\{ \begin{array}{l} \mathbb{F} \in {}^m\mathbb{K}_m \\ \mathbb{F} \in {}^n\mathbb{K}_n \end{array} \right. \quad \mathbb{F} = \mathbb{F}^* \begin{array}{c|ccc} \lambda_1^{1/2} & 0 & 0 & \\ \hline 0 & \cdot & 0 & \\ \hline 0 & 0 & \lambda_r^{1/2} & \\ \hline 0 & 0 & 0 & \\ \hline 0 & 0 & 0 & \\ \hline 0 & 0 & 0 & \end{array} \mathbb{F}$$

$${}^m\mathbb{K}_m \ni \mathbb{F} \mathbb{F}^* \text{ self-adj} \Rightarrow \left\{ \begin{array}{l} \bigvee \mathbb{F} = \begin{bmatrix} 1 \\ \cdot \\ m \end{bmatrix} \\ {}^i\mathbb{F} \mathbb{F} \mathbb{F}^* = \lambda_i {}^i\mathbb{F} \end{array} \right. \subset_{\text{ONB}} \mathbb{K}_m \Rightarrow \left({}^i\mathbb{F} \mathbb{F} \right) \left({}^j\mathbb{F} \mathbb{F} \right)^* = {}^i\mathbb{F} \left(\mathbb{F} \mathbb{F}^* \right) {}^j\mathbb{F} = \lambda_i {}^i\delta_j \Rightarrow \lambda_i \geq 0$$

$$\lambda_1 \geq \dots \geq \lambda_r > 0 = \lambda_{r+1} = \dots = \lambda_m$$

$$r = \text{rank } \mathbb{F} \leq m \wedge n$$

$$\Rightarrow \begin{bmatrix} \lambda_1^{-1/2} \\ \cdot \\ \lambda_r^{-1/2} \end{bmatrix} \subset_{\text{ONS}} \mathbb{K}_n$$

$$\Rightarrow \bigvee \mathbb{F} = \begin{bmatrix} \lambda_1^{-1/2} \\ \cdot \\ \lambda_r^{-1/2} \\ \cdot \\ n \end{bmatrix} \subset_{\text{ONB}} \mathbb{K}_n \Rightarrow \mathbb{F} \mathbb{F} = \begin{bmatrix} 1 \\ \cdot \\ r \\ 0 \\ \cdot \\ 0 \end{bmatrix} = \begin{array}{c|ccc} \lambda_1^{1/2} & 0 & 0 & \\ \hline 0 & \cdot & 0 & \\ \hline 0 & 0 & \lambda_r^{1/2} & \\ \hline 0 & 0 & 0 & \\ \hline 0 & 0 & 0 & \\ \hline 0 & 0 & 0 & \end{array} \mathbb{F}$$

$$\mathbb{K} = \mathbb{R}:\mathbb{C}:\mathbb{H}$$

$$\Gamma \begin{array}{c} \textcircled{3} \\ \textcircled{0} \end{array} \Gamma$$

$$n \mathbb{K}_n^{\mathfrak{D}}$$