$$\begin{split} \mathbf{K}_{n} \stackrel{\blacktriangleright}{\vdash} &= \frac{\mathbf{\Gamma} = \mathbf{K}_{n}}{\dim \mathbf{\Gamma} = 1} = \frac{\mathbf{K}\mathbf{\Gamma}}{0 \neq \mathbf{\Gamma} \in {}^{1}\mathbf{K}_{n}} \\ & \operatorname{cpt} {}^{1}\mathbf{K}_{n}^{U} \xrightarrow{\operatorname{surj}} \mathbf{K}_{n} \stackrel{\blacktriangleright}{\vdash} \operatorname{quo-top} \operatorname{cpt} \\ & \mathbf{K}_{n} \stackrel{\blacktriangleright}{\vdash} \supset U_{i} = \frac{\mathbf{K}\left[\mathbf{\Gamma}_{1} \ \cdots \ \mathbf{\Gamma}_{n}\right]}{\mathbf{\Gamma}_{i} \neq 0} \\ \\ & \mathbf{K}_{n} \stackrel{\blacktriangleright}{\vdash} \supset U_{i} \xrightarrow{\mathbf{V}_{i}} \stackrel{i}{\underset{\mathbf{K}_{n-i}}{\vdash} \in \mathbf{K} \\ & \mathbf{K}\left[\mathbf{\Gamma}_{1} \ \cdots \ \mathbf{\Gamma}_{n}\right] \mapsto \left[\frac{\mathbf{\Gamma}_{1}}{\mathbf{\Gamma}_{i}} \ \frac{\mathbf{\Gamma}_{i}}{\mathbf{\Gamma}_{i}} \ \frac{\mathbf{\Gamma}_{n}}{\mathbf{\Gamma}_{i}}\right] \\ & \mathbf{K}\left[\mathbf{\Gamma}_{1} \ 1 \ \mathbf{\Gamma}_{n}\right] \mapsto \left[\mathbf{\Gamma}_{1} \ \mathbf{\Gamma}_{i} \ \mathbf{\Gamma}_{n}\right] \\ & \mathbf{K}\left[\mathbf{\Gamma}_{1} \ 1 \ \mathbf{\Gamma}_{n}\right] \leftrightarrow \left[\mathbf{\Gamma}_{1} \ \mathbf{\Gamma}_{i} \ \mathbf{\Gamma}_{n}\right] \\ & \mathbf{K}\left[\mathbf{\Gamma}_{1} \ 1 \ \mathbf{\Gamma}_{n}\right] = \mathbf{K}\left[\frac{\mathbf{\Gamma}_{1}}{\mathbf{\Gamma}_{i}} \ \frac{\mathbf{\Gamma}_{n}}{\mathbf{\Gamma}_{i}}\right] \Rightarrow \mathbf{\mathcal{V}}_{i} \mathbf{\mathcal{V}}_{i} = I|U_{i} \\ & \left[\mathbf{\Gamma}_{1} \ \mathbf{\Gamma}_{n}\right] \mathbf{\mathcal{V}}_{i} = \left[\frac{\mathbf{\Gamma}_{1}}{\mathbf{\Gamma}_{i}} \ \frac{\mathbf{\Gamma}_{n}}{\mathbf{\Gamma}_{i}}\right] = \left[\mathbf{\Gamma}_{1} \ \mathbf{\Gamma}_{n}\right] \Rightarrow \mathbf{\mathcal{V}}_{i} \mathbf{\mathcal{V}}_{i} = I \end{aligned}$$

$$\begin{cases} U_i \stackrel{r_i}{\cap}_{i} = \begin{bmatrix} n \\ 1 \end{bmatrix} & \text{Atlas} \\ \mathbf{K}_n \stackrel{p^*}{\leftarrow} \in \mathbb{K}_n \end{cases}$$
overdeck
$$K\Gamma \in \prod_n \stackrel{p^*}{\to} \Gamma \neq 0 \Rightarrow \bigvee_{i \in \begin{bmatrix} n \\ 1 \end{bmatrix}} \Gamma_i \neq 0 \Rightarrow \mathbb{K}\Gamma \in U_i$$

$$\text{overlap } i \neq j \xrightarrow{r_i} i < j$$

$$U_i \cap U_j = \frac{\mathbb{K} \begin{bmatrix} \Gamma_1 & \cdots & \Gamma_n \end{bmatrix}}{\Gamma_i \neq 0 \neq \Gamma_j}$$

$$U_i \cap U_j [\mathcal{V}_i \in \stackrel{i}{\mathbb{K}_{n-i}}$$

$$U_i \cap U_j [\mathcal{V}_i \in \stackrel{i}{\mathbb{K}_{n-i}}]$$

$$U_i \cap U_j [\mathcal{V}_i \in \stackrel{i}{\mathbb{K}_{n-i}}]$$

$$U_i \cap U_j [\mathcal{V}_i \in \stackrel{j}{\mathbb{K}_{n-i}}]$$

$$\begin{bmatrix} \Gamma_i & \hat{\Gamma}_i & \Gamma_j \neq 0 & \Gamma_n \end{bmatrix} \mapsto \begin{bmatrix} \frac{\Gamma_i}{\Gamma_j} & \frac{1}{\Gamma_j} & \frac{\Gamma_j}{\Gamma_j} & \frac{\Gamma_i}{\Gamma_j} \end{bmatrix}$$

$$\underbrace{\mathbb{K} \begin{bmatrix} \Gamma_1 & \cdots & \Gamma_n \end{bmatrix}}_{V_i \stackrel{i}{\to} J_j} \stackrel{i}{\to} J_j = \begin{bmatrix} \frac{\Gamma_i}{\Gamma_i} & \frac{\Gamma_i}{\Gamma_j} & \frac{\Gamma_i}{\Gamma_j} & \frac{\Gamma_i}{\Gamma_j} & \frac{\Gamma_i}{\Gamma_j} & \frac{\Gamma_i}{\Gamma_j} \end{bmatrix} = \begin{bmatrix} \frac{\Gamma_i}{\Gamma_j} & \frac{\Gamma_i}{\Gamma_j} & \frac{\Gamma_i}{\Gamma_j} & \frac{\Gamma_i}{\Gamma_j} & \frac{\Gamma_i}{\Gamma_j} \end{bmatrix} = \underbrace{\mathbb{K} \begin{bmatrix} \Gamma_1 & \cdots & \Gamma_n \end{bmatrix}}_{V_i \stackrel{i}{\to} V_i \stackrel{i}$$