

$$\Delta/4 = \partial_z \bar{\partial}_z = \partial_i \bar{\partial}_i$$

$$e_\alpha \mathfrak{K} = \exp \frac{1}{2\nu\tilde{\alpha}} \partial_i \bar{\partial}_i = \exp \frac{1}{8\nu\tilde{\alpha}} \Delta$$

$$\underbrace{e_\alpha \mathfrak{K}}_z x \mathbf{e}_z {}^z \mathbf{e}_y = x \mathbf{e}_y^{1/2\nu\tilde{\alpha}} x \mathbf{e}_z {}^z \mathbf{e}_y$$

$$\begin{aligned} \text{LHS} &= \left(\frac{2\nu\tilde{\alpha}}{\pi} \right)^d \int_{d\zeta}^{\mathbb{C}^d} z^{-\zeta} \mathbf{e}_{z-\zeta}^{-\tilde{\alpha}} x \mathbf{e}_\zeta {}^\zeta \mathbf{e}_y = \tilde{\alpha}^d x \mathbf{e}_z {}^z \mathbf{e}_y \left(\frac{2\nu}{\pi} \right)^d \int_{d\zeta}^{\mathbb{C}^d} \zeta^{-z} \mathbf{e}_{\zeta-z}^{-\tilde{\alpha}} x \mathbf{e}_{\zeta-z} {}^{\zeta-z} \mathbf{e}_y \\ &= \tilde{\alpha}^d \tilde{\alpha}^{-d} x \mathbf{e}_z {}^z \mathbf{e}_y x \mathbf{e}_y^{1/2\nu\tilde{\alpha}} = \text{RHS} \end{aligned}$$

$$\partial_i \bar{\partial}_i x \mathbf{e}_z {}^z \mathbf{e}_y = \underbrace{\bar{\partial}_i x}_{\mathbf{e}_z} \underbrace{\partial_i {}^z \mathbf{e}_y}_{\mathbf{e}_y} = \underbrace{(x|e_i)}_x \mathbf{e}_z \underbrace{(e_i|y)}_y {}^z \mathbf{e}_y = (x|y) x \mathbf{e}_z {}^z \mathbf{e}_y$$

$$\exp t \partial_i \bar{\partial}_i x \mathbf{e}_z {}^z \mathbf{e}_y = x \mathbf{e}_y^t x \mathbf{e}_z {}^z \mathbf{e}_y \Rightarrow t = \frac{1}{2\nu\tilde{\alpha}}$$