$$\text{⅃} \xleftarrow[\text{nilp}]{\Gamma} \text{⅃} \Longrightarrow \in|\text{⅃} \xleftarrow[\text{nilp}]{\Gamma\times} \in|\text{⅃}$$

$$\Gamma \underset{\text{nilp}}{\ltimes} \text{⅃} \Longrightarrow \bigwedge_{i \geqslant j} {}^{i}\Gamma_{j} = 0$$

$$\text{Ƒ} \text{ k-nilp} \Longleftrightarrow \bigwedge_{i-j \geqslant k} {}^{i}\Gamma_{j} = 0$$

$$\text{Ƒ} \text{ k-nilp} \Longrightarrow \Gamma \times \text{Ƒ} \text{ k-1 nilp}$$

$$p - q \geqslant k-1 \Longrightarrow q+k-p \leqslant 1 \Longrightarrow$$

$${}^{p}\underbrace{\Gamma \times \text{Ƒ}}_{q} = {}^{p}\Gamma_{\ell}\, {}^{\ell}\text{Ƒ}_{q} - {}^{p}\text{Ƒ}_{j}\, {}^{j}\Gamma_{q} = \overbrace{\sum_{p<\ell}\sum_{\ell<q+k}}^{p<\ell<q+k \Rightarrow q+k-p\leqslant 2} {}^{p}\Gamma_{\ell}\, {}^{\ell}\text{Ƒ}_{q} - \overbrace{\sum_{p<j+k}\sum_{j<q}}^{p-k<j<q \Rightarrow q+k-p \leqslant 2} {}^{p}\text{Ƒ}_{j}\, {}^{j}\Gamma_{q} = 0$$

$$\text{Ƒ} \text{ r+1 nilp} \Longrightarrow \Gamma \times \text{Ƒ} \text{ r-nilp} \Longrightarrow \Gamma \overset{2}{\times} \text{Ƒ} \text{ r-1 nilp} \Longrightarrow \cdots \Longrightarrow \Gamma \overset{r}{\times} \text{ 0 nilp}$$

$$\mathfrak{g} \underset{\text{nilp}}{\ltimes} V \Rightarrow \backslash^{\mathfrak{g}} V \neq 0$$

$$\text{Ind } d = \dim \mathfrak{g} \geqslant 0$$

$$0 \leqslant d-1 \mapsto d: \quad \dim \mathfrak{g} = d$$

$$\mathfrak{h} \underset{\text{subalg}}{\overset{\max}{\sqsubset}} \mathfrak{g} \Rightarrow \mathfrak{h} \underset{\text{nilp}}{\ltimes} V \underset{\dim \mathfrak{h} < d}{\Rightarrow} \backslash^{\mathfrak{h}} V \neq 0$$

$$\mathfrak{g} \neg \mathfrak{h} \xleftarrow[\text{nilp}]{\mathfrak{h} \times} \mathfrak{g} \neg \mathfrak{h}$$

$$\mathfrak{h} \times \underbrace{x + \mathfrak{h}} = \underbrace{\mathfrak{h} \times x} + \mathfrak{h}$$

$$\Rightarrow \bigvee 0 \neq x + \mathfrak{h} \in \backslash^{\mathfrak{h}} \mathfrak{g} \neg \mathfrak{h} \neq 0 \Rightarrow x \in \mathfrak{g} \llcorner \mathfrak{h}$$

$$x \times \mathfrak{g} \subset \mathfrak{h} \Rightarrow \mathfrak{h} \subset x\mathbb{K} + \mathfrak{h} \underset{\text{subalg}}{\sqsubset} \mathfrak{h} \underset{\max}{\Rightarrow} x\mathbb{K} + \mathfrak{h} = \mathfrak{h} \Rightarrow \dim \mathfrak{g} \neg \mathfrak{h} = 1$$

$$\Rightarrow \mathfrak{g} \times \mathfrak{h} = \underbrace{x\mathbb{K} + \mathfrak{h}} \times \mathfrak{h} \subset x \times \mathfrak{h} + \mathfrak{h} \times \mathfrak{h} \subset \mathfrak{h} \Rightarrow \mathfrak{h} \underset{\text{ideal}}{\sqsubset} \mathfrak{g}$$

$$\backslash^{\mathfrak{h}} V \xleftarrow[\text{nilp}]{x \ltimes} \backslash^{\mathfrak{h}} V$$

$$\bigwedge_{V}^{V} \bigwedge_{\mathfrak{h}}^{\mathfrak{h}} \mathfrak{h} \ltimes \underbrace{x \ltimes V} = x \ltimes \underbrace{\mathfrak{h} \ltimes V}_{=0} + \underbrace{\overbrace{\mathfrak{h} \times x}^{\in \mathfrak{h}} \ltimes V}_{=0} = 0 \Rightarrow x \ltimes V \in \backslash^{\mathfrak{h}} V$$

$$x \underset{\text{nilp}}{\ltimes} \backslash^{\mathfrak{h}} V \neq 0 \Rightarrow 0 \neq \backslash^{x} \backslash^{\mathfrak{h}} V = \backslash^{\mathfrak{g}} V$$