

$$\begin{array}{c} \mathbb{C}^{1|1} \diagup \mathbb{C} \\ \downarrow \overline{\langle \rangle} \\ \Psi | \mathbb{C}^{1|1} \diagdown \mathbb{C} \end{array}$$

quant /Geod <sub>$\alpha$</sub> / Toep / Weyl

$$\text{allg } \overline{\zeta^{00}\mathbb{J} + \zeta^{10}\mathbb{J} + \bar{\zeta}^{01}\mathbb{J} + \bar{\zeta}\zeta^{11}\mathbb{J}} = \frac{a\overline{\mathbb{J}} + \alpha\overline{\mathbb{J}}}{c\overline{\mathbb{J}}} \Bigg| \frac{b\overline{\mathbb{J}}}{d\overline{\mathbb{J}} + \delta\overline{\mathbb{J}}}$$

$$\frac{\overbrace{^0\!\!0\mathbb{J} + \zeta^{10}\mathbb{J} + \bar{\zeta}^{01}\mathbb{J} + \bar{\zeta}\zeta^{11}\mathbb{J}}^{\zeta^{10}\mathbb{J} + \bar{\zeta}^{01}\mathbb{J} + \bar{\zeta}\zeta^{11}\mathbb{J}}}{(1-\alpha)\nu^{\overbrace{10\mathbb{J}}^{00\mathbb{J}}} + \bar{\zeta}\zeta^{11}\mathbb{J}} = \frac{(1-\alpha)\nu^{\overbrace{00\mathbb{J}}^{00\mathbb{J}}} + \overbrace{\zeta^{11}\mathbb{J}}^{11\mathbb{J}}}{(1-\alpha)\nu^{\overbrace{10\mathbb{J}}^{00\mathbb{J}}}} \Bigg| \frac{(1-\alpha)\overbrace{\zeta^{01}\mathbb{J}}^{01\mathbb{J}}}{(1-\alpha)\nu^{\overbrace{00\mathbb{J}}^{00\mathbb{J}}} + \alpha\overbrace{\zeta^{11}\mathbb{J}}^{11\mathbb{J}}}$$

$$\begin{aligned}
& z|\zeta \widehat{\overline{\mathbb{J}}\mathbb{I}} = \int_{d^2w/\pi}^{\mathbb{C}^{1|0}} \int_{d^2\eta}^{\mathbb{C}^{0|1}} w|\eta \mathbb{J} \widehat{z|\zeta \overline{w|\eta}\mathbb{I}} = \int_{d^2w/\pi}^{\mathbb{C}^{1|0}} \int_{d^2\eta}^{\mathbb{C}^{0|1}} z-w|\zeta - \eta \mathfrak{e}_{w|\eta}^{(1-\alpha)\nu} w|\eta \mathbb{J}^{\alpha z + (1-\alpha)w|\alpha\zeta + (1-\alpha)\eta\mathbb{I}} \\
&= \int_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w \mathfrak{e}_w^{\nu(1-\alpha)} \int_{d^2\eta}^{\mathbb{C}^{0|1}} \zeta - \eta \mathfrak{e}_\eta^{(1-\alpha)\nu} \underbrace{^0\!\!0\mathbb{J} + \eta^{10}\mathbb{J} + \bar{\eta}^{01}\mathbb{J} + \bar{\eta}\eta^{11}\mathbb{J}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} \\
&\quad = \int_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w \mathfrak{e}_w^{\nu(1-\alpha)} \int_{d^2\eta}^{\mathbb{C}^{0|1}} \underbrace{1 + (1-\alpha)\nu(\zeta\bar{\eta} + \bar{\eta}\eta)}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} \\
&\quad \underbrace{^0\!\!0\mathbb{J} + \eta^{10}\mathbb{J} + \bar{\eta}^{01}\mathbb{J} + \bar{\eta}\eta^{11}\mathbb{J}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} \\
&= \int_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w \mathfrak{e}_w^{\nu(1-\alpha)} \begin{cases} (1-\alpha)\nu^0_w \underbrace{\mathbb{J}^{\alpha z + (1-\alpha)w_0\mathbb{I} + \alpha\zeta^{\alpha z + (1-\alpha)w_1\mathbb{I}} + (1-\alpha)\eta^{\alpha z + (1-\alpha)w_1\mathbb{I}}}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} \\ + (1-\alpha)\nu^{\overbrace{10\mathbb{J}}^{00\mathbb{J}}} \underbrace{\mathbb{J}^{\alpha z + (1-\alpha)w_0\mathbb{I} + \alpha\zeta^{\alpha z + (1-\alpha)w_1\mathbb{I}}}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} \\ + (1-\alpha)\nu^{\overbrace{01\mathbb{J}}^{10\mathbb{J}}} \underbrace{\mathbb{J}^{\alpha z + (1-\alpha)w_0\mathbb{I}}}_{\alpha z + (1-\alpha)w_0\mathbb{I}} \\ + (1-\alpha)\nu^{\overbrace{11\mathbb{J}}^{11\mathbb{J}}} \underbrace{\mathbb{J}^{\alpha z + (1-\alpha)w_0\mathbb{I} + \alpha\zeta^{\alpha z + (1-\alpha)w_1\mathbb{I}}}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} \end{cases} \\
&= (1-\alpha) \underbrace{z \widehat{^0\!\!1\mathbb{J}}_1\mathbb{I}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} + \underbrace{z \widehat{^1\!\!1\mathbb{J}}_0\mathbb{I}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} + \alpha\zeta \underbrace{z \widehat{^1\!\!1\mathbb{J}}_1\mathbb{I}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} + (1-\alpha)^2 \nu\zeta \underbrace{z \widehat{^0\!\!0\mathbb{J}}_1\mathbb{I}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} \\
&\quad + (1-\alpha)\nu\zeta \underbrace{z \widehat{^0\!\!1\mathbb{J}}_0\mathbb{I}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} + (1-\alpha)\nu \underbrace{z \widehat{^0\!\!0\mathbb{J}}_0\mathbb{I}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}} + (1-\alpha)\alpha\nu\zeta \underbrace{z \widehat{^0\!\!0\mathbb{J}}_1\mathbb{I}}_{\alpha z + (1-\alpha)w_0\mathbb{I} + \overbrace{\alpha\zeta + (1-\alpha)\eta}^{\alpha z + (1-\alpha)w_1\mathbb{I}}}
\end{aligned}$$

$$\boxed{\begin{array}{c|c} \overline{\mathbb{J}} & \stackrel{\text{glob}}{=} \frac{\nu^{\overline{00}\mathbb{J}} + \overline{\nu^{11}\mathbb{J}}}{\nu^{\overline{10}\mathbb{J}}} \\ \hline & \nu^{\overline{01}\mathbb{J}} \\ & \nu^{\overline{00}\mathbb{J}} \end{array}}$$

$$\begin{aligned}
z|\zeta \widehat{\overline{\mathbb{J}}\mathbb{I}} &= \int_{d^2w/\pi}^{\mathbb{C}^{1|0}} \int_{d^2\eta}^{\mathbb{C}^{0|1}} w|\eta \mathbb{J} z|\zeta \widehat{w|\eta \mathbb{I}} = \int_{d^2w/\pi}^{\mathbb{C}^{1|0}} \int_{d^2\eta}^{\mathbb{C}^{0|1}} z-w|\zeta-\eta \mathbf{e}_w^\nu w|\eta \mathbb{J}^{w|\eta} \mathbb{I} \\
&= \int_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w \mathbf{e}_w^\nu \int_{d^2\eta}^{\mathbb{C}^{0|1}} \zeta-\eta \mathbf{e}_\eta^\nu \underbrace{\mathbb{J}^{00} + \eta^{10}\mathbb{J}}_{\mathbb{J}^{01}} + \underbrace{\bar{\eta}^{01}\mathbb{J} + \bar{\eta}\eta^{11}\mathbb{J}}_{\mathbb{J}^{10}} \underbrace{\mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{w|\eta}} \\
&= \int_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w \mathbf{e}_w^\nu \int_{d^2\eta}^{\mathbb{C}^{0|1}} \underbrace{1 + \nu\zeta\bar{\eta} + \nu\bar{\eta}\eta}_{\mathbb{J}^{00}} \underbrace{\mathbb{J}^{00} + \eta^{10}\mathbb{J}}_{\mathbb{J}^{01}} + \underbrace{\bar{\eta}^{01}\mathbb{J} + \bar{\eta}\eta^{11}\mathbb{J}}_{\mathbb{J}^{10}} \underbrace{\mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{w|\eta}} \\
&= \int_{d^2w/\pi}^{\mathbb{C}^{1|0}} z-w \mathbf{e}_w^\nu \underbrace{\mathbb{J}^{01} \mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{01}\mathbb{J}^{w|\eta}\mathbb{I}} + \underbrace{\mathbb{J}^{11} \mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{11}\mathbb{J}^{w|\eta}\mathbb{I}} + \nu\zeta \underbrace{\mathbb{J}^{00} \mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{00}\mathbb{J}^{w|\eta}\mathbb{I}} + \nu^{10} \mathbb{J}^{w|\eta} \mathbb{I} \\
&= \underbrace{\mathbb{J}^{01}\mathbb{J}^{w|\eta}\mathbb{I}}_{\mathbb{J}^{01}\mathbb{J}^{w|\eta}\mathbb{I}} + \underbrace{\mathbb{J}^{11}\mathbb{J}^{w|\eta}\mathbb{I}}_{\mathbb{J}^{11}\mathbb{J}^{w|\eta}\mathbb{I}} + \nu\zeta \underbrace{\mathbb{J}^{00}\mathbb{J}^{w|\eta}\mathbb{I}}_{\mathbb{J}^{00}\mathbb{J}^{w|\eta}\mathbb{I}} + \nu^{10} \mathbb{J}^{w|\eta} \mathbb{I} \\
&\Leftarrow \widehat{\overline{\mathbb{J}}\mathbb{I}} = \int_{\nu d^2w/\pi}^{\mathbb{C}} z-w \mathbf{e}_w^\nu w \mathbb{J}^{w|\eta} \mathbb{I} \\
w|\omega \widehat{\overline{\mathbb{J}}\mathbb{I}} &= \overbrace{\mathbb{J}^{00} + \omega^{10}\mathbb{J} + \bar{\omega}^{01}\mathbb{J} + \omega\bar{\omega}^{11}\mathbb{J}}_{\mathbb{J}^{00}\mathbb{J}^{10}\mathbb{J}^{01}\mathbb{J}^{11}\mathbb{I}} \underbrace{\mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{w|\eta}\mathbb{I}} \\
&= \mathbb{J}^{00} \mathbb{J}^{w|\eta} \mathbb{I} + \omega \overbrace{\mathbb{J}^{00} \mathbb{J}^{w|\eta} \mathbb{I} + \mathbb{J}^{10} \mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{00}\mathbb{J}^{10}\mathbb{J}^{w|\eta}\mathbb{I}} + \bar{\omega} \mathbb{J}^{01} \mathbb{J}^{w|\eta} \mathbb{I} + \omega\bar{\omega} \overbrace{\mathbb{J}^{11} \mathbb{J}^{w|\eta} \mathbb{I} - \mathbb{J}^{01} \mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{11}\mathbb{J}^{01}\mathbb{J}^{w|\eta}\mathbb{I}} \\
\Rightarrow z|\zeta \widehat{\overline{\mathbb{J}}\mathbb{I}} &= z|\zeta \widehat{\mathcal{P}_\nu(\overline{\mathbb{J}}\mathbb{I})} = \underbrace{z \widehat{P_\nu^{00} \mathbb{J}_0 \mathbb{I}}}_{\mathbb{J}^{00}\mathbb{J}^{w|\eta}\mathbb{I}} + \zeta \underbrace{z \widehat{P_\nu^{00} \mathbb{J}_1 \mathbb{I} + P_\nu^{10} \mathbb{J}_0 \mathbb{I}}}_{\mathbb{J}^{00}\mathbb{J}^{11}\mathbb{J}^{01}\mathbb{J}^{w|\eta}\mathbb{I}} - \frac{1}{\nu} \underbrace{z \widehat{P_\nu^{11} \mathbb{J}_0 \mathbb{I} - P_\nu^{01} \mathbb{J}_1 \mathbb{I}}}_{\mathbb{J}^{11}\mathbb{J}^{00}\mathbb{J}^{w|\eta}\mathbb{I}} \\
&= \underbrace{\mathbb{J}^{00} \mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{00}\mathbb{J}^{w|\eta}\mathbb{I}} - \frac{1}{\nu} \underbrace{\mathbb{J}^{11} \mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{11}\mathbb{J}^{w|\eta}\mathbb{I}} + \frac{1}{\nu} \underbrace{\mathbb{J}^{01} \mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{01}\mathbb{J}^{w|\eta}\mathbb{I}} + \zeta \underbrace{\mathbb{J}^{10} \mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{10}\mathbb{J}^{w|\eta}\mathbb{I}} + \zeta \underbrace{\mathbb{J}^{00} \mathbb{J}^{w|\eta} \mathbb{I}}_{\mathbb{J}^{00}\mathbb{J}^{w|\eta}\mathbb{I}}
\end{aligned}$$

$$\overline{\zeta^{00}\mathbb{J} + \zeta^{10}\mathbb{J} + \bar{\zeta}^{01}\mathbb{J} + \bar{\zeta}\zeta^{11}\mathbb{J}} \stackrel{\text{glob}}{\underset{\text{Weyl}}{=}} \frac{2\nu^{\overline{00}\mathbb{J}} + \overline{11}\mathbb{J}}{2\nu^{\overline{10}\mathbb{J}}} \quad \left| \quad \begin{array}{c} 2\overline{01}\mathbb{J} \\ 2\nu^{\overline{00}\mathbb{J}} - \overline{11}\mathbb{J} \end{array} \right.$$

$$\begin{aligned}
& z|\zeta \widehat{\overline{\mathbb{J}\gamma}} = \int_{d^2 w/\pi}^{\mathbb{C}^{1|0}} \int_{d^2 \eta}^{\mathbb{C}^{0|1}} {}_{w|\eta} \mathbb{J} \widehat{\overline{w|\eta}\gamma} = \int_{d^2 w/\pi}^{\mathbb{C}^{1|0}} \int_{d^2 \eta}^{\mathbb{C}^{0|1}} z-w|\zeta - \omega \mathfrak{e}_{w|\eta}^{2\nu} {}_{w|\eta} \mathbb{J}^{2w-z} \Big|_{\eta} - \zeta \gamma \\
& = \int_{d^2 w/\pi}^{\mathbb{C}^{1|0}} z-w \mathfrak{e}_w^{2\nu} \int_{d^2 \eta}^{\mathbb{C}^{0|1}} \zeta - \eta \mathfrak{e}_\eta^{2\nu} \underbrace{\underline{\eta^0 \mathbb{J}} + \eta^1 \mathbb{J} + \bar{\eta}^0 \mathbb{J} + \bar{\eta}^1 \mathbb{J}}_{\underline{2w-z}} \underbrace{\underline{0 \gamma} + \widehat{2\eta - \zeta}^{2w-z} \underline{1 \gamma}}_{\underline{0 \gamma}} \\
& = \int_{d^2 w/\pi}^{\mathbb{C}^{1|0}} z-w \mathfrak{e}_w^{2\nu} \int_{d^2 \eta}^{\mathbb{C}^{0|1}} \underbrace{1 + 2\nu\zeta\bar{\eta} + 2\nu\bar{\eta}\eta}_{\underline{w^0 \mathbb{J}}} + \eta^1 \mathbb{J} + \bar{\eta}^0 \mathbb{J} + \bar{\eta}\eta^1 \mathbb{J} \underbrace{\underline{2w-z} \underline{0 \gamma} - \zeta^{2w-z} \underline{1 \gamma} + 2\eta^{2w-z} \underline{1 \gamma}}_{\underline{0 \gamma}} \\
& = \int_{d^2 w/\pi}^{\mathbb{C}^{1|0}} z-w \mathfrak{e}_w^{2\nu} \begin{cases} 2\nu_w^0 \mathbb{J}^{2w-z} \underline{0 \gamma} - \zeta^{2w-z} \underline{1 \gamma} + 2\zeta^{2w-z} \underline{1 \gamma} \\ 2\nu\zeta_w^0 \mathbb{J}^{2w-z} \underline{0 \gamma} - \zeta^{2w-z} \underline{1 \gamma} \\ 2_w^{01} \mathbb{J}^{2w-z} \underline{1 \gamma} \\ 2_w^{11} \mathbb{J}^{2w-z} \underline{0 \gamma} - \zeta^{2w-z} \underline{1 \gamma} \end{cases} \\
& = \int_{d^2 w/\pi}^{\mathbb{C}^{1|0}} z-w \mathfrak{e}_w^{2\nu} \underbrace{\underline{11 \mathbb{J}} + 2\nu_w^0 \mathbb{J}^{2w-z} \underline{0 \gamma} + \zeta 2\nu_w^1 \mathbb{J}^{2w-z} \underline{0 \gamma} + 2_w^{01} \mathbb{J}^{2w-z} \underline{1 \gamma} + \zeta \underline{2\nu_w^0 \mathbb{J}} - \underline{11 \mathbb{J}}^{2w-z} \underline{1 \gamma}}_{\underline{0 \gamma}} \\
& = \int_{d^2 w/\pi}^{\mathbb{C}^{1|0}} z-w \mathfrak{e}_w^{2\nu} \underbrace{\underline{11 \mathbb{J}} + 2\nu_w^0 \mathbb{J}^{2w-z} \underline{0 \gamma}}_{\underline{0 \gamma}} + 2 \int_{d^2 w/\pi}^{\mathbb{C}^{1|0}} z-w \mathfrak{e}_w^{2\nu} \underbrace{\underline{01 \mathbb{J}}^{2w-z} \underline{1 \gamma}}_{\underline{0 \gamma}} \\
& + 2\nu\zeta \int_{d^2 w/\pi}^{\mathbb{C}^{1|0}} z-w \mathfrak{e}_w^{2\nu} \underbrace{\underline{10 \mathbb{J}}^{2w-z} \underline{0 \gamma}}_{\underline{0 \gamma}} + \zeta \int_{d^2 w/\pi}^{\mathbb{C}^{1|0}} z-w \mathfrak{e}_w^{2\nu} \underbrace{\underline{2\nu_w^0 \mathbb{J}} - \underline{11 \mathbb{J}}^{2w-z} \underline{1 \gamma}}_{\underline{0 \gamma}} \\
& = \underbrace{z \widehat{\overline{11 \mathbb{J} + 2\nu_w^0 \mathbb{J}}_0 \gamma}}_{\underline{0 \gamma}} + 2 \underbrace{z \widehat{\overline{01 \mathbb{J}}_1 \gamma}}_{\underline{0 \gamma}} + 2\nu\zeta \underbrace{z \widehat{\overline{10 \mathbb{J}}_0 \gamma}}_{\underline{0 \gamma}} + 2\nu\zeta \underbrace{z \widehat{\overline{00 \mathbb{J} - 11 \mathbb{J}}_1 \gamma}}_{\underline{0 \gamma}} \\
& \Leftarrow z \widehat{\overline{\mathbb{J}\gamma}} = \int_{d^2 w/\pi}^{\mathbb{C}} z-w \mathfrak{e}_w^{2\nu} {}_w \mathbb{J}^{2w-z} \gamma
\end{aligned}$$