

$$\mathbb{F} \text{ alg abg : } \mathbb{F} \underset{\text{ideal}}{\subseteq} \mathbb{F}^n \underset{\bullet}{\triangleleft} \mathbb{F} \xrightarrow[\text{Hilbert}]{\text{strong}} \underbrace{\mathbb{F}^n \underset{\mathbb{F}}{\subseteq}}_{\mathbb{F}} \underset{\bullet}{\triangleleft} \mathbb{F} = \sqrt{\mathbb{F}}$$

$$\text{noeth } \mathbb{F}^n \underset{\bullet}{\triangleleft} \mathbb{F} \Rightarrow \mathbb{F} = \langle \gamma^\alpha \rangle \underset{\bullet}{\triangleleft} \mathbb{F}$$

$$\mathbb{F}^n \underset{\mathbb{F}}{\subseteq} \underset{\text{ex}}{\subseteq} \mathbb{F}^n$$

$$1 \in \underbrace{\mathbb{F}^n \underset{\mathbb{F}}{\subseteq}}_{\mathbb{F}} \underset{\bullet}{\triangleleft} \mathbb{F} \Rightarrow \underbrace{\mathbb{F}^n \underset{\mathbb{F}}{\subseteq}}_{\mathbb{F}} \Big/ 1 = 0$$

$$\mathbb{F} = \langle X\gamma^\alpha : 1 - X\gamma Z \rangle \underset{\bullet}{\triangleleft} \mathbb{F}^{1+n}$$

$$\mathbb{F}^{1+n} \underset{\mathbb{F}}{\subseteq} = \emptyset$$

$$\zeta \bigvee a_0 : a \in \mathbb{F}^{1+n} \underset{\mathbb{F}}{\subseteq} \Rightarrow \begin{cases} a\gamma^\alpha = 0 \Rightarrow a \in \mathbb{F}^n \underset{\mathbb{F}}{\subseteq} & \Rightarrow a\gamma = 0 \\ a_0 a\gamma = 1 & \Rightarrow a\gamma \neq 0 \end{cases} \zeta$$

$$\xrightarrow[\text{Hilb}]{\text{weak}} \mathbb{F} = \langle X\gamma^\alpha : 1 - X\gamma Z \rangle \underset{\bullet}{\triangleleft} \mathbb{F}^{1+n} = \mathbb{F}^{1+n} \underset{\bullet}{\triangleleft} \mathbb{F} \Rightarrow$$

$$1 = X\gamma^\alpha X\alpha\gamma^Z + \underbrace{1 - X\gamma Z}_{X\gamma^Z} = X\gamma^\alpha X\alpha\gamma_j Z^j + \underbrace{1 - X\gamma Z}_{X\gamma_k Z^k} = \underbrace{X\gamma^\alpha X\alpha\gamma_0 + X\gamma_0}_{=1} + \underbrace{X\gamma^\alpha X\alpha\gamma_{k+1} + X\gamma_{k+1} - X\gamma X\gamma_k}_{=0} Z^k$$

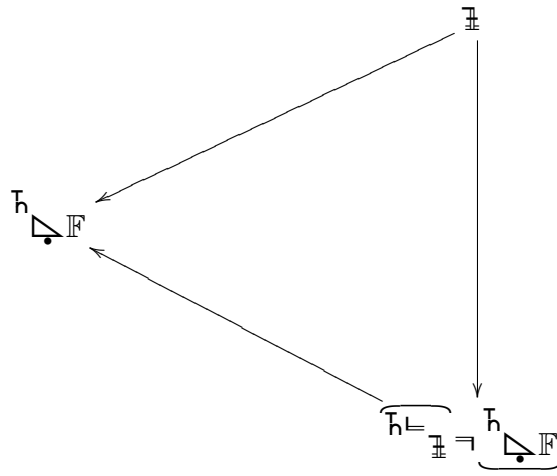
$$d = \underbrace{\deg X\alpha\gamma}_\gamma \vee \underbrace{\deg X\gamma}_{+1}$$

$$X\gamma^d = \underbrace{X\gamma^\alpha X\alpha\gamma_0 + X\gamma_0}_{=1} X\gamma^d + \underbrace{X\gamma^\alpha X\alpha\gamma_{k+1} + X\gamma_{k+1} - X\gamma X\gamma_k}_{=0} X\gamma^{d-1-k}$$

$$= \underbrace{X\gamma^\alpha X\alpha\gamma_0 X\gamma^d + X\alpha\gamma_{k+1} X\gamma^{d-1-k}}_{\in \mathbb{F}} + X\gamma_0 X\gamma^d + \underbrace{X\gamma_{k+1} X\gamma^{d-1-k} - X\gamma X\gamma_k X\gamma^{d-1-k}}_{=-X\gamma_0 X\gamma^d} \in \mathbb{F} \Rightarrow X\gamma \in \sqrt{\mathbb{F}}$$

$$\mathbb{F} \underset{\bullet}{\triangleleft} \ni \mathfrak{h} \Rightarrow \mathfrak{h} \underset{\bullet}{\triangleleft} \mathbb{F} \in \overset{\epsilon}{\triangleleft} \mathbb{F} \text{ abel elg}$$

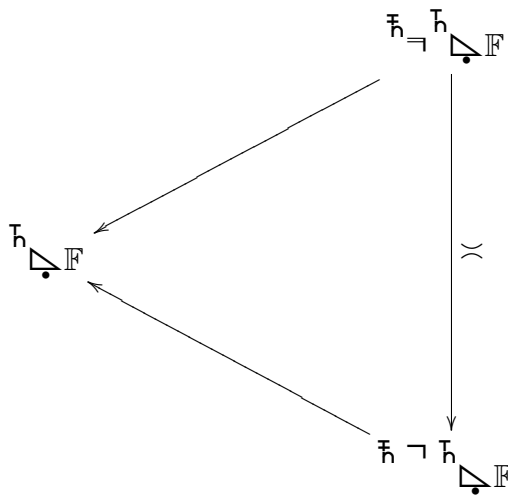
$$\mathbb{F} \underset{\text{ex}}{\subseteq} \mathfrak{h} \underset{\bullet}{\triangleleft} \mathbb{F}$$



$$\text{Rad } \mathbb{H} = \overline{\mathfrak{h}}_{\mathbb{H}} \supset \mathfrak{h}_{\bullet} \mathbb{F}$$

$$\mathfrak{h} \subset \mathfrak{h} = \mathbb{F} \triangleleft \overline{\mathfrak{h}}_{\mathfrak{h}_{\bullet} \mathbb{F}}$$

$$\mathfrak{h}_{\supset} \mathfrak{h}_{\bullet} \mathbb{F} = \frac{\gamma \in \mathfrak{h}_{\bullet} \mathbb{F}}{\bigwedge_{\mathfrak{h}} \mathfrak{h} \gamma = 0}$$



$$\mathfrak{h}_{i \Rightarrow} \mathfrak{h}_{i \Rightarrow} \mathbb{F} \supset \mathfrak{h}_{i \Rightarrow} \mathfrak{h}_{i \Rightarrow} \mathbb{F} \Leftrightarrow \mathfrak{h} \subset \mathfrak{h}$$

$$\bigcup_i \mathfrak{h}_{i \Rightarrow} \mathfrak{h}_{i \Rightarrow} \mathbb{F} = \bigcap_i \mathfrak{h}_{i \Rightarrow} \mathfrak{h}_{i \Rightarrow} \mathbb{F}$$

$$\bigwedge_x \bigvee_{\ell} a^{\ell} x \in \mathfrak{m}$$

$$\mathfrak{m} \subset \frac{x \in R}{\bigvee_{\ell} a_r^{\ell} x \in \mathfrak{m}} \stackrel{\text{ideal}}{\sqsubseteq} R$$

$$\xrightarrow{\text{max}} \frac{x \in R}{\bigvee_{\ell} a_r^{\ell} x \in \mathfrak{m}} = R$$

$$a_r \notin \mathfrak{m} \Rightarrow \mathfrak{p}^* + \mathfrak{m}R(X) \neq R(X)$$

$$\nexists \mathfrak{p}^* + \mathfrak{m}R(X) = R(X) \Rightarrow \bigvee_f^{\mathfrak{p}^*} \bigvee_{i^c}^{\mathfrak{m}} 1 = {}^X f + \sum_i {}^{i^c} X^i \xrightarrow{\text{eucl}} \underbrace{{}^X f - 1}_{\in \mathfrak{m}} X^{r-1} = {}^X g {}^X h + \sum_r^j b_j X^j \Rightarrow \bigwedge_r^j \bigvee_{\ell_j}^{\mathbb{N}} a_r^{\ell_j} b_j \in \mathfrak{m}$$

$$\ell = \max_j \ell_j \Rightarrow \bigwedge_j^r a_r^{\ell} b_j \in \mathfrak{m} \Rightarrow a_r^{\ell} \underbrace{{}^X f - 1}_{\in \mathfrak{m}} X^{r-1} = a_r^{\ell} {}^X g {}^X h + \sum_r^j a_r^{\ell} b_j X^j$$

$$\Rightarrow \underbrace{a_r^{\ell} + a_r^{\ell} b_{r-1}}_{\in \mathfrak{m}} X^{r-1} + \sum_{r-1}^j a_r^{\ell} b_j X^j = a_r^{\ell} X^{r-1} + \sum_r^j a_r^{\ell} b_j X^j = a_r^{\ell} {}^X f X^{r-1} + a_r^{\ell} {}^X g {}^X h \in \mathfrak{p}^*$$

$$a_r^{\ell} \notin \mathfrak{m} \ni a_r^{\ell} b_{r-1} \Rightarrow a_r^{\ell} + a_r^{\ell} b_{r-1} \neq 0 \nexists$$