$$\det x_j^{r\,-\,i} = \overbrace{\begin{array}{|c|c|c|c|}\hline 1 & x_0 & \dots & x_0^n \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline 1 & x_n & \dots & x_n^n \\ \hline \end{array}} = \prod_{0 \leqslant i < j \leqslant n} \overbrace{x_j - x_i}$$

$$\overbrace{\begin{array}{|c|c|c|c|c|c|}\hline 1 & x_0 & x_0^2 & \dots & x_0^{n-1} & x_0^n \\ \hline 1 & x_1 & x_1^2 & \dots & x_1^{n-1} & x_1^n \\ \hline \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline 1 & x_{n-1} & x_{n-1}^2 & \dots & x_{n-1}^{n-1} & x_{n-1}^n \\ \hline 1 & x_n & x_n^2 & \dots & x_n^{n-1} & x_n^n \\ \hline \end{array}} \underset{\mathrm{Spa}_j - x_0 \mathrm{Spa}_{j-1}}{\overset{\mathrm{Spa}_j \mapsto}{=}} \overbrace{\begin{array}{|c|c|c|c|c|c|}\hline 1 & x_0 - x_0 & x_0^2 - x_0 x_0 & \dots & x_0^{n-1} - x_0 x_0^{n-2} & x_0^n - x_0 x_0^{n-1} \\ \hline 1 & x_1 - x_0 & x_1^2 - x_0 x_1 & \dots & x_1^{n-1} - x_0 x_1^{n-2} & x_1^n - x_0 x_1^{n-1} \\ \hline \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline 1 & x_{n-1} - x_0 & x_{n-1}^2 - x_0 x_{n-1} & \dots & x_{n-1}^{n-1} - x_0 x_{n-1}^{n-2} & x_{n-1}^n - x_0 x_{n-1}^{n-1} \\ \hline 1 & x_n - x_0 & x_n^2 - x_0 x_n & \dots & x_n^{n-1} - x_0 x_n^{n-2} & x_n^n - x_0 x_n^{n-1} \\ \hline \end{array}} =$$

$$\overbrace{\begin{array}{|c|c|c|c|c|c|}\hline 1 & 0 & 0 & \dots & 0 & 0 \\ \hline 1 & x_1 - x_0 & \overbrace{x_1 - x_0} x_1 & \dots & \overbrace{x_1 - x_0} x_1^{n-2} & \overbrace{x_1 - x_0} x_1^{n-1} \\ \hline \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline 1 & x_{n-1} - x_0 & \overbrace{x_{n-1} - x_0} x_{n-1} & \dots & \overbrace{x_{n-1} - x_0} x_{n-1}^{n-2} & \overbrace{x_{n-1} - x_0} x_{n-1}^{n-1} \\ \hline 1 & x_n - x_0 & \overbrace{x_n - x_0} x_n & \dots & \overbrace{x_n - x_0} x_n^{n-2} & \overbrace{x_n - x_0} x_n^{n-1} \\ \hline \end{array}} = \prod_{1 \leqslant j \leqslant n} \overbrace{x_j - x_0} \overbrace{\begin{array}{|c|c|c|c|c|c|}\hline 1 & 0 & 0 & \dots & 0 & 0 \\ \hline \overbrace{x_1 - x_0}^{-1} & 1 & x_1 & \dots & x_1^{n-2} & x_1^{n-1} \\ \hline \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline \overbrace{x_{n-1} - x_0}^{-1} & 1 & x_{n-1} & \dots & x_{n-1}^{n-2} & x_{n-1}^{n-1} \\ \hline \overbrace{x_n - x_0}^{-1} & 1 & x_n & \dots & x_n^{n-2} & x_n^{n-1} \\ \hline \end{array}}$$

$$= \prod_{0 < j \leqslant n} \overbrace{x_j - x_0} \overbrace{\begin{array}{|c|c|c|c|c|}\hline 1 & x_1 & \dots & x_1^{n-2} & x_1^{n-1} \\ \hline \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline 1 & x_{n-1} & \dots & x_{n-1}^{n-2} & x_{n-1}^{n-1} \\ \hline 1 & x_n & \dots & x_n^{n-2} & x_n^{n-1} \\ \hline \end{array}} \underset{\text{ind}}{=} \prod_{0 < j \leqslant n} \overbrace{x_j - x_0} \prod_{1 \leqslant i < j \leqslant n} \overbrace{x_j - x_i} = \prod_{0 \leqslant i < j \leqslant n} \overbrace{x_j - x_i}$$