

$$\frac{(2\pi)^{d-r}}{\Gamma_{d/r}^X} \frac{\Gamma_{1+a/2}^r}{\Gamma_{1+ra/2}} \int_{dx_i}^{0 \uparrow 1} x \Phi^{\varkappa} \prod_i x_i^\alpha \overbrace{1-x_i}^\beta \prod_{i < j} \overbrace{x_j-x_i}^a = \frac{\Gamma_{\varkappa+\alpha+d/r}^X \Gamma_{\beta+d/r}^X}{\Gamma_{\varkappa+\alpha+\beta+2d/r}^X}$$

$$f_{\varkappa}^{a/2} = \prod_{i < j} \frac{\Gamma_{\varkappa_i - \varkappa_j + (j-i)a/2}}{\Gamma_{\varkappa_i - \varkappa_j + (j-i)a/2}}$$

$$r! f_0^{a/2} = r! \prod_{i < j} \frac{\Gamma_{(j-i)a/2}}{\Gamma_{(j-i)a/2}} = r! \prod_i \frac{\Gamma_{(r+1-i)a/2}}{\Gamma_{a/2}} = \frac{\prod_k k \Gamma_{ka/2}}{\Gamma_{a/2}^r} = \frac{\prod_k \Gamma_{ka/2}^{ka/2}}{\Gamma_{a/2}^r (a/2)^r} = \frac{\prod_k \Gamma_{1+ka/2}}{\Gamma_{1+a/2}^r}$$

$$= \frac{\Gamma_{1+ra/2}}{\Gamma_{1+a/2}^r} \prod_i \Gamma_{1+(r-i)a/2} = \frac{\Gamma_{1+ra/2}}{\Gamma_{1+a/2}^r} \frac{\Gamma_{d/r}^X}{\sqrt{2\pi}^{d-r}}$$

$$e_{S_{\varkappa}^{a/2}} \stackrel{\text{KAD}}{=} f_{\varkappa}^{a/2} / f_0^{a/2}$$

$$\frac{1}{\sqrt{2\pi}^{d-r}} \frac{\Gamma_{\varkappa+\alpha+d/r}^X \Gamma_{\beta+d/r}^X}{\Gamma_{\varkappa+\alpha+\beta+2d/r}^X} = \prod_i \frac{\Gamma_{\varkappa_i+\alpha+d/r-(i-1)a/2} \Gamma_{\beta+d/r-(i-1)a/2}}{\Gamma_{\varkappa_i+\alpha+\beta+2d/r-(i-1)a/2}}$$

$$= \prod_i \frac{\Gamma_{\varkappa_i+\alpha+1+(r-i)a/2} \Gamma_{\beta+1+(r-i)a/2}}{\Gamma_{\varkappa_i+\alpha+\beta+2+(2r-i-1)a/2}} \stackrel{\text{KAD}}{=} \frac{1}{r! f_{\varkappa}^{a/2}} \int_{dx}^{0 \uparrow 1} x S_{\varkappa}^{a/2} \prod_i x_i^\alpha \overbrace{1-x_i}^\beta \prod_{i < j} \overbrace{x_i-x_j}^a$$

$$= \frac{1}{r! f_0^{a/2}} \int_{dx}^{0 \uparrow 1} x \Phi^{\varkappa} \prod_i x_i^\alpha \overbrace{1-x_i}^\beta \prod_{i < j} \overbrace{x_i-x_j}^a = \frac{\Gamma_{1+a/2}^r \sqrt{2\pi}^{d-r}}{\Gamma_{1+ra/2} \Gamma_{d/r}^X} \int_{dx}^{0 \uparrow 1} x \Phi^{\varkappa} \prod_i x_i^\alpha \overbrace{1-x_i}^\beta \prod_{i < j} \overbrace{x_i-x_j}^a$$

$$f_0^{a/2} = \prod_i \frac{\Gamma_{ra/2-(i-1)a/2}}{\Gamma_{a/2}} = \frac{\Gamma_{ra/2}^X}{\Gamma_{a/2}^r \sqrt{2\pi}^{d-r}}$$

$$P_{\varkappa}(e) = \prod_{i:j \in \varkappa} \frac{r+1-i+\frac{a}{2}(j-1)}{\varkappa_j^\sharp+1-i+\frac{a}{2}(\varkappa_i-j)} = \prod_{i:j \in \varkappa} \frac{\frac{a}{2}(r+1-i)+j-1}{\frac{a}{2}(\varkappa_j^\sharp+1-i)+\varkappa_i-j}$$