$$\mathbb{K}^{2^{p:q}}$$

$$\eta_K = \prod_{k \in K} \eta_{kk}$$

$$\lrcorner \ni {}_i\lrcorner \quad \text{ONB}$$

$$\eta_{i:j} = {}_i\lrcorner \veebar {}_j\lrcorner$$

$$I\sharp J = \prod_{I \ni i \neq j \in J} i\sharp j \, \eta_{I \cap J} = \overset{I}{\underset{J}{\vee}} \eta_{I \cap J}$$

$$I + J := (I \cup J) \llcorner (I \cap J) = (I \llcorner J) \cup (J \llcorner I)$$

$$\chi_{I+J} = \chi_I + \chi_J : N \xrightarrow{\text{Add in 2}}$$

$$(I + J) + K = I + (J + K)$$

$$\chi_{\text{LHS}} = \chi_{I+J} + \chi_K = \left(\chi_I + \chi_J\right) + \chi_K = \chi_I + \left(\chi_J + \chi_K\right) = \chi_I + \chi_{J+K} = \chi_{\text{RHS}}$$

$$(I + J)\sharp K = I\sharp K J\sharp K = I\sharp (J + K)$$

$$\overset{I+J}{\underset{K}{\vee}} \eta_{\underbrace{I+J} \cap K} = \overset{I}{\underset{K}{\vee}} \eta_{I \cap K} \overset{J}{\underset{K}{\vee}} \eta_{J \cap K} = \overset{I}{\underset{J+K}{\vee}} \eta_{I \cap \underbrace{J+K}}$$

$$\text{LHS} = \prod_{\ell \in I+K} \ell\sharp K \, \eta_{(I+J) \cap K} = \frac{\prod_{i \in I} i\sharp K}{\prod_{i \in I \cap J} i\sharp K} \frac{\prod_{j \in J} j\sharp K}{\prod_{j \in J \cap I} j\sharp K} \frac{\eta_{I \cap K}}{\eta_{I \cap J \cap K}} \frac{\eta_{J \cap K}}{\eta_{J \cap I \cap K}} = I\sharp K \, \eta_{I \cap K} J\sharp K \, \eta_{J \cap K} = I\sharp K J\sharp K$$

$$\text{LHS} = \prod_{\ell \in I+K} \overset{\ell}{\underset{K}{\vee}} \eta_{(I+J) \cap K} = \frac{\prod_{i \in I} \overset{i}{\underset{K}{\vee}}}{\prod_{i \in I \cap J} \overset{i}{\underset{K}{\vee}}} \frac{\prod_{j \in J} \overset{j}{\underset{K}{\vee}}}{\prod_{j \in J \cap I} \overset{j}{\underset{K}{\vee}}} \frac{\eta_{I \cap K}}{\eta_{I \cap J \cap K}} \frac{\eta_{J \cap K}}{\eta_{J \cap I \cap K}} = \overset{I}{\underset{K}{\vee}} \eta_{I \cap K} \eta_{I \cap K} \overset{J}{\underset{K}{\vee}} \eta_{J \cap K} \eta_{J \cap K}$$

$${}_I\lrcorner \times {}_J\lrcorner = I\sharp J \;\; {}_{I+J}\lrcorner = \overset{I}{\underset{J}{\vee}} \eta_{I \cap J} \;\; {}_{I+J}\lrcorner$$

$${}_\bullet\lrcorner \times {}_J\lrcorner = \bullet\sharp J \;\; {}_{\bullet+J}\lrcorner = {}_J\lrcorner = {}_J\lrcorner \times {}_\bullet\lrcorner \Rightarrow {}_\bullet\lrcorner = 1$$

$${}_\bullet\lrcorner \times {}_J\lrcorner = \overset{\bullet}{\underset{J}{\wedge}} \eta_{\bullet \cap J} \;\; {}_{\bullet+J}\lrcorner = {}_J\lrcorner = {}_J\lrcorner \times {}_\bullet\lrcorner \Rightarrow {}_\bullet\lrcorner = 1$$

$$\overbrace{{}_I\lrcorner \times {}_J\lrcorner} \times {}_K\lrcorner = {}_I\lrcorner \times \overbrace{{}_J\lrcorner \times {}_K\lrcorner}$$

$$\text{LHS} = I\sharp J \;\; {}_{I+J}\lrcorner \times {}_K\lrcorner = I\sharp J \, (I+J)\sharp K \;\; {}_{\underbrace{I+J}+K}\lrcorner = I\sharp J I\sharp K J\sharp K \;\; {}_{I+\underbrace{J+K}}\lrcorner$$

$$= J\sharp K I\sharp \underbrace{J+K} \;\; {}_{I+\underbrace{J+K}}\lrcorner = J\sharp K \;\; {}_I\lrcorner \times {}_{J+K}\lrcorner = \text{RHS}$$

$$\text{LHS} = \overset{I}{\underset{J}{\vee}} \eta_{I \cap J} \;\; {}_{I+J}\lrcorner \times {}_K\lrcorner = \overset{I}{\underset{J}{\vee}} \eta_{I \cap J} \overset{I+J}{\underset{K}{\vee}} \eta_{\underbrace{I+J} \cap K} \;\; {}_{\underbrace{I+J}+K}\lrcorner = \overset{I}{\underset{J}{\vee}} \eta_{I \cap J} \overset{I}{\underset{K}{\vee}} \eta_{I \cap K} \overset{J}{\underset{K}{\vee}} \eta_{J \cap K} \;\; {}_{I+\underbrace{J+K}}\lrcorner$$

$$= \bigvee_{K}^{J} \eta_{J \cap K} \bigvee_{J+K}^{I} \eta_{I \cap \underbrace{J+K}} {}_{I+\underbrace{J+K}}\daleth = \bigvee_{K}^{J} \eta_{J \cap K} {}_{I}\daleth \times {}_{J+K}\daleth = \text{ RHS}$$

$${}_{I}\daleth = {}_{i_1}\daleth \times \cdots {}_{i_k}\daleth$$

$$I = \underbrace{e} t\, i_1 < \cdots < i_k$$

$${}_{i}\daleth \times {}_{j}\daleth = i\sharp j \;{}_{i+j}\daleth = \begin{cases} \eta^{ii} {}_{\bullet}\daleth & i = j \\ i\sharp j \;{}_{ij}\daleth = -j\sharp i \;{}_{ji}\daleth = -{}_{j}\daleth \times {}_{i}\daleth & i \neq j \end{cases}$$

$${}_{I}\daleth \times {}_{J}\daleth = -1^{\overline{I||J| - |I \cap J}} \;{}_{J}\daleth \times {}_{I}\daleth$$

$${}_{I}\bar{\daleth} := -1^{\bar{I}} \;{}_{I}\daleth$$

$${}_{\bullet}\bar{\daleth} = -1^{\bar{\bullet}} = {}_{\bullet}\daleth$$

$${}_{j}\bar{\daleth} = -1^{1} \;{}_{j}\daleth = - \;{}_{j}\daleth$$

$$\underline{{}_{I}\daleth \times {}_{J}\daleth} = {}_{I}\bar{\daleth} \times {}_{J}\bar{\daleth}$$

$$\text{LHS } = \left|{}^{I} \sharp J \;{}_{I+J}\daleth \right| = -1^{I+J} \overbrace{I \sharp J} \;{}_{I+J}\daleth = -1^{I+J} \;{}_{I}\daleth \times {}_{J}\daleth = -1^{\overline{I| + |J| - 2|I \cap J}} \;{}_{I}\daleth \times {}_{J}\daleth = -1^{\bar{I}} -1^{\bar{J}} \;{}_{I}\daleth \times {}_{J}\daleth = \text{ RHS}$$

$$\text{LHS } = \underline{\bigvee_{J}^{I} \eta_{I \cap J} {}^{I+J}\daleth} = -1^{I+J} \overbrace{\bigvee_{J}^{I} \eta_{I \cap J}} \;{}_{I+J}\daleth = -1^{I+J} \;{}_{I}\daleth \times {}_{J}\daleth = -1^{\overline{I| + |J| - 2|I \cap J}} \;{}_{I}\daleth \times {}_{J}\daleth = -1^{\bar{I}} -1^{\bar{J}} \;{}_{I}\daleth \times {}_{J}\daleth = \text{ RHS}$$

$${}_{I}\bar{\bar{\daleth}} = -1^{\bar{I}} \;{}_{I}\bar{\daleth} = {}_{I}\daleth$$

$${}_{j}\daleth \times {}_{I}\bar{\daleth} = -1^{\bar{I}} \;{}_{j}\daleth \times {}_{I}\daleth = \begin{cases} {}_{I}\daleth \times {}_{j}\daleth & j \notin I \text{ vert } |I| \text{ mal} \\ -{}_{I}\daleth \times {}_{j}\daleth & j \in I \text{ vert } |I| - 1 \text{ mal} \end{cases}$$

$$\mathbb{L} \in \mathbb{K}_{\mathbb{I}} \triangleleft \daleth$$

$$\bigwedge_{\mathsf{L} \in \daleth} \mathbb{L} \times \mathsf{L} = \mathsf{L} \times \bar{\mathbb{L}} \Rightarrow \mathbb{L} \in \mathbb{K}$$

$$\mathbb{L} = \sum_{I} \mathbb{L}^{I} \;{}_{I}\daleth \Rightarrow \bigwedge_{j \in N} \sum_{j \in I} \mathbb{L}^{I} \;{}_{I}\daleth \times {}_{j}\daleth + \sum_{j \notin I} \mathbb{L}^{I} \;{}_{I}\daleth \times {}_{j}\daleth = \mathbb{L} \times {}_{j}\daleth = {}_{j}\daleth \times \bar{\mathbb{L}} = \sum_{I} \mathbb{L}^{I} \;{}_{j}\daleth \times {}_{I}\bar{\daleth} = -\sum_{j \in I} \mathbb{L}^{I} \;{}_{I}\daleth \times {}_{j}\daleth +$$

$$\Rightarrow 0 = 2 \overbrace{\sum_{j \in I} \mathbb{L}^{I} {}_{I}\daleth \times} {}_{j}\overset{\text{inv}}{\daleth} \Rightarrow 0 = \sum_{j \in I} \mathbb{L}^{I} \;{}_{I}\daleth \Rightarrow \bigwedge_{j} \bigwedge_{I \ni j} \mathbb{L}^{I} = 0 \Rightarrow \bigwedge_{I \neq \varnothing} \mathbb{L}^{I} = 0$$

$${}_{I}\daleth^{t} = -1^{m} \;{}_{I}\daleth$$

$$|I| - 2m \in 2$$

$$1^{t} = {}_{\bullet}\daleth^{t} = -1^{0} \;{}_{\bullet}\daleth = {}_{\bullet}\daleth = 1$$

$$ {}_{i}\mathtt{l}^{t} = -1^{0} \; {}_{i}\mathtt{l} = {}_{i}\mathtt{l} $$

$$ \overbrace{{}_{I}\mathtt{l} \times {}_{J}\mathtt{l}}^{t} = {}_{J}\mathtt{l}^{t} \times {}_{I}\mathtt{l}^{t} $$

$$ \text{LHS} = I \sharp j \; {}_{I+J}\mathtt{l}^{t} = I \sharp \mathbb{1} \; l^{m_{I+J}} \; {}_{I+J}\mathtt{l} = -1^{m_{I+J}} \; {}_{I}\mathtt{l} \times {}_{J}\mathtt{l} $$

$$ \text{LHS} = \bigwedge_{j}^{I} \eta_{I \cap j} \; {}_{I+J}\mathtt{l}^{t} = \bigvee_{J}^{I} \eta_{I \cap J} \; -1^{m_{I+J}} \; {}_{I+J}\mathtt{l} = -1^{m_{I+J}} \; {}_{I}\mathtt{l} \times {}_{J}\mathtt{l} $$

$$ \text{RHS} = -1^{m_J + m_I} \; {}_{J}\mathtt{l} \times {}_{I}\mathtt{l} = -1^{m_I + m_J + \overline{IJ} - \overline{I \cap J}} \; {}_{I}\mathtt{l} \times {}_{J}\mathtt{l} $$

$$ \overline{I+J} - 2\left(m_I + m_J + \overline{IJ} - \overline{I \cap J}\right) = \underbrace{\overline{I+J} + 2\overline{I \cap J}}_{= \overline{I} + \overline{J}} - 2\overline{IJ} - 2m_I - 2m_J = \underbrace{\overline{I} - 2m_I}_{\in 2} + \underbrace{\overline{J} - 2m_J}_{\in 2} - 2\overline{IJ} $$

$$ \implies m_I + m_J + \overline{IJ} - \overline{I \cap J} * m_{\overline{I+J}} $$