$$\underset{\pm}{\mathbb{R}} \triangledown \mathbb{R}^{8k+r:8\ell+s} = {}_{2^{4(k+\ell)}}\left(\underset{\pm}{\mathbb{R}} \triangledown \mathbb{R}^{r:s}\right)^{2^{4(k+\ell)}}$$

$$r - s \textcircled{c} 0, 6 \Longrightarrow {}_{2^{(r+s)/2}}\mathbb{R}^{2^{(r+s)/2}}$$

$$\dim_{\mathbb{R}} = \left(2^{(r+s)/2}\right)^2 = 2^{2(r+s)/2} = 2^{r+s}$$

$$r - s \textcircled{c} 1, 5 \Longrightarrow {}_{2^{(r+s-1)/2}}\mathbb{C}^{2^{(r+s-1)/2}}$$

$$\dim_{\mathbb{R}} = 2 \cdot \left(2^{(r+s-1)/2}\right)^2 = 2 \cdot 2^{2(r+s-1)/2} = 2 \cdot 2^{r+s-1} = 2^{r+s}$$

$$r - s \textcircled{c} 2, 4 \Longrightarrow {}_{2^{(r+s-2)/2}}\mathbb{H}^{2^{(r+s-2)/2}}$$

$$\dim_{\mathbb{R}} = 4 \cdot \left(2^{(r+s-2)/2}\right)^2 = 4 \cdot 2^{2(r+s-2)/2} = 4 \cdot 2^{r+s-2} = 2^{r+s}$$

$$r - s \textcircled{c} 7 \Longrightarrow {}_{2^{(r+s-1)/2}}\mathbb{R}^{2^{(r+s-1)/2}} \mathbin{⧗} {}_{2^{(r+s-1)/2}}\mathbb{R}^{2^{(r+s-1)/2}}$$

$$\dim_{\mathbb{R}} = 2 \cdot \left(2^{(r+s-1)/2}\right)^2 = 2 \cdot 2^{2(r+s-1)/2} = 2 \cdot 2^{r+s-1} = 2^{r+s}$$

$$r - s \textcircled{c} 3 \Longrightarrow {}_{2^{(r+s-3)/2}}\mathbb{H}^{2^{(r+s-3)/2}} \mathbin{⧗} {}_{2^{(r+s-3)/2}}\mathbb{H}^{2^{(r+s-3)/2}}$$

$$\dim_{\mathbb{R}} = 2 \cdot 4 \cdot \left(2^{(r+s-3)/2}\right)^2 = 2 \cdot 4 \cdot 2^{2(r+s-3)/2} = 2 \cdot 4 \cdot 2^{r+s-3} = 2^{r+s}$$

Clerc's list

$$2^2: \quad {}_{2^1}\mathbb{R}^{2^1}$$

$$2^3: \quad {}_{2^1}\mathbb{C}^{2^1}$$

$$2^4: \quad {}_{2^1}\mathbb{H}^{2^1}$$

$$2^5: \quad {}_{2^1}\mathbb{H}^{2^1} \mathbin{⧗} {}_{2^1}\mathbb{H}^{2^1}$$

$$2^6: \quad {}_{2^2}\mathbb{H}^{2^2}$$

$$2^7: \quad {}_{2^3}\mathbb{C}^{2^3}$$

$$2^8: \quad {}_{2^4}\mathbb{R}^{2^4}$$

$$2^9: \quad {}_{2^4}\mathbb{R}^{2^4} ⧗ {}_{2^4}\mathbb{R}^{2^4}$$

spin factor representations

$\mathbb{R}_2$

$\mathbb{C}_2$

$\mathbb{H}_2$

$+\mathbb{H}_2$

$\mathbb{H}_4$

$\mathbb{C}_8$

$\mathbb{R}_{16}$

$+\mathbb{R}_{16}$

$\mathbb{R}_{32}$

$\mathbb{C}_{32}$

$\mathbb{R}_k \to \mathbb{R}_k \times \mathbb{R}_k \to \mathbb{R}_{2k}$

$\mathbb{H}_k \to \mathbb{H}_k \times \mathbb{H}_k \to \mathbb{H}_{2k}$

$\mathbb{R}_k \to \mathbb{C}_k \to \mathbb{H}_k$ add β_{neu}

$\mathbb{H}_k \to \mathbb{C}_{2k} \to \mathbb{R}_{4k}$