$$n = \{0 \cdot\cdot n-1\}$$

$${}_n\mathbb{1} \text{ central } \Leftrightarrow n \text{ odd}$$

$${}_i\mathbb{1}\ {}_C\mathbb{1} = \begin{cases} \overset{|C|}{-1}\ {}_C\mathbb{1}\ {}_i\mathbb{1} & i \notin C \\ \overset{|C|-1}{-1}\ {}_C\mathbb{1}\ {}_i\mathbb{1} & i \in C \end{cases}$$

$${}_i\mathbb{1}\ {}_n\mathbb{1} = \overset{n-1}{-1}\ {}_n\mathbb{1}\ {}_i\mathbb{1}$$

$${}_n\mathbb{1} \times {}_n\mathbb{1} = -1^{q+n(n-1)/2} = \begin{cases} -1^q & 4\mathbb{N} \ni n \in 4\mathbb{N}+1 \\ -1^{q+1} & 4\mathbb{N}+2 \ni n \in 4\mathbb{N}+3 \end{cases}$$

$${}_n\overset{2}{\mathbb{1}} = {}_0\mathbb{1} \times {}_1\mathbb{1} \times \cdots \times {}_{n-1}\mathbb{1} \times {}_0\mathbb{1} \times {}_1\mathbb{1} \times \cdots \times {}_{n-1}\mathbb{1}$$

$$= -1^{n(n-1)/2}\ {}_0\overset{2}{\mathbb{1}} \times {}_1\overset{2}{\mathbb{1}} \times \cdots \times {}_{n-1}\overset{2}{\mathbb{1}} = -1^{n(n-1)/2}\ 1^{\not 1}\ l^q$$

$$n = 4k+\ell \Longrightarrow \frac{n\,\overbrace{n-1}}{2} = \frac{\overbrace{4k+\ell}\,\overbrace{4k+\ell-1}}{2} = 2k\underbrace{4k+\ell-1} + 2\ell k + \frac{\ell\,\overbrace{\ell-1}}{2} \underset{\text{mod } 2}{\asymp} \frac{\ell\,\overbrace{\ell-1}}{2} = \begin{cases} 0 & \ell = 0 \\ 0 & \ell = 1 \\ 1 & \ell = 2 \\ 3 & \ell = 3 \end{cases}$$

$${}_n\mathbb{1} \text{ central symmetry } \Leftrightarrow p - q \in 4\mathbb{Z}+1$$

$$\text{even } q = 2\ell \Longrightarrow p+q = n = 4k+1 \Longrightarrow p-q = p+q-2q = 4k+1-4\ell = 4(k-\ell)+1 \in 4\mathbb{Z}+1$$

$$\text{odd } q = 2\ell+1 \Longrightarrow p+q = n = 4k+3 \Longrightarrow p-q = p+q-2q = 4k+3-4\ell-2 = 4(k-\ell)+1 \in 4\mathbb{Z}+1$$

$$n \text{ odd } \wedge\ {}_n\mathbb{1} \times {}_n\mathbb{1} = 1 \Longrightarrow 1 \neq {}_n\mathbb{1} \text{ central symmetry } \Rightarrow \frac{1 \pm {}_n\mathbb{1}}{2} \text{ orth central proj}$$

$$\mathbb{K} \lhd \mathbb{L} \text{ einf } \Leftrightarrow p - q \notin 4\mathbb{Z} + 1$$

$$\mathbb{K} \lhd \mathbb{L} \sqsupseteq_{\text{id}} \mathcal{I} \ni \tilde{\mathbb{L}} = \sum_{A \subset n} \tilde{\mathbb{L}}^{A} {}_{A}\mathbb{L} \neq 0$$

$$\text{OE } \tilde{\mathbb{L}}^{B} = 1$$

$$\mathcal{I} \ni \mathbb{L} = \tilde{\mathbb{L}} \times {}_{B}\mathbb{L}^{-1} = \sum_{A \subset n} \tilde{\mathbb{L}}^{A} {}_{A}\mathbb{L} \times {}_{B}\mathbb{L}^{-1} = 1 + \sum_{B \neq A \subset n} \tilde{\mathbb{L}}^{A} {}_{A}\mathbb{L} \times {}_{B}\mathbb{L}^{-1} = 1 + \sum_{\varnothing \neq C \subset n} \mathbb{L}^{C} {}_{C}\mathbb{L}$$

$$\mathcal{I} \ni \begin{cases} 1 & n \text{ even} \\ 1 + \mathbb{L}^{n} {}_{n}\mathbb{L} & n \text{ odd} \end{cases}$$

$${}_{A}\mathbb{L} \, {}_{B}\mathbb{L} = \pm \, {}_{B}\mathbb{L} \, {}_{A}\mathbb{L}$$

$$\mathcal{I} \ni {}_{0}\mathbb{L} = \frac{\mathbb{L} + {}_{0}\mathbb{L}\mathbb{L}\,{}_{0}\mathbb{L}^{-1}}{2} = 1 + \sum_{\varnothing \neq C \subset n} \mathbb{L}^{C} \frac{{}_{C}\mathbb{L} + {}_{0}\mathbb{L}\,{}_{C}\mathbb{L}\,{}_{0}\mathbb{L}^{-1}}{2} = 1 + \sum_{\varnothing \neq C \subset n}^{{}_{0}\mathbb{L}\,{}_{C}\mathbb{L} = {}_{C}\mathbb{L}\,{}_{0}\mathbb{L}} \mathbb{L}^{C} {}_{C}\mathbb{L}$$

$$\mathcal{I} \ni {}_{1}\mathbb{L} = \frac{{}_{0}\mathbb{L} + {}_{1}\mathbb{L}\,{}_{0}\mathbb{L}\,{}_{1}\mathbb{L}^{-1}}{2} = 1 + \sum_{\varnothing \neq C \subset n}^{{}_{0}\mathbb{L}\,{}_{C}\mathbb{L} = {}_{C}\mathbb{L}\,{}_{0}\mathbb{L}} \mathbb{L}^{C} \frac{{}_{C}\mathbb{L} + {}_{1}\mathbb{L}\,{}_{C}\mathbb{L}\,{}_{1}\mathbb{L}^{-1}}{2} = 1 + \sum_{\varnothing \neq C \subset n}^{\begin{bmatrix} {}_{0}\mathbb{L}\,{}_{C}\mathbb{L} = {}_{C}\mathbb{L}\,{}_{0}\mathbb{L} \\ {}_{1}\mathbb{L}\,{}_{C}\mathbb{L} = {}_{C}\mathbb{L}\,{}_{1}\mathbb{L} \end{bmatrix}} \mathbb{L}^{C} {}_{C}\mathbb{L}$$

$$\Longrightarrow \cdots \Longrightarrow \mathcal{I}_{+} \ni \sum_{\varnothing \neq C \subset n}^{{}_{i}\mathbb{L}\,{}_{C}\mathbb{L} = {}_{C}\mathbb{L}\,{}_{i}\mathbb{L}} \mathbb{L}^{C} {}_{C}\mathbb{L} = \begin{cases} 1 & n \text{ even } \Leftrightarrow {}_{n}\mathbb{L} \text{ not central} \\ 1 + \mathbb{L}^{n} {}_{n}\mathbb{L} & n \text{ odd } \Leftrightarrow {}_{n}\mathbb{L} \text{ central} \end{cases}$$

$$n \text{ even } \Longrightarrow 1 \in \mathcal{I} \Longrightarrow \mathbb{K} \lhd \mathbb{L} \text{ einf}$$

$$n \text{ odd } \wedge {}_{n}\mathbb{L} \times {}_{n}\mathbb{L} = -1 \Longrightarrow \mathcal{I} \ni \underbrace{1 + \mathbb{L}^{n} {}_{n}\mathbb{L}}\,\underbrace{1 - \mathbb{L}^{n} {}_{n}\mathbb{L}} = 1 - \mathbb{L}^{2n} {}_{n}\mathbb{L}^{2} = 1 + \mathbb{L}^{2n} > 0 \Longrightarrow \mathbb{K} \lhd \mathbb{L} \text{ einf}$$