$$\frac{a \ \bigg|\ \mathsf{L}}{\mathsf{L}^T \ \bigg|\ b} = a\left(\frac{1}{2}{:}\frac{i}{2}{:}0\right) + b\left(\frac{1}{2}{:}-\frac{i}{2}{:}0\right) + (0{:}0{:}\mathsf{L}) = \left(\frac{a+b}{2}{:}i\frac{a-b}{2}{:}\mathsf{L}\right): \quad a \in \mathbb{R} \ni b{:}\mathsf{L} \in i\mathbb{R}^a$$

$$(\alpha{:}\beta{:}\mathsf{L}) = \frac{\alpha - i\beta \ \bigg|\ \mathsf{L}}{\mathsf{L}^T \ \bigg|\ \alpha + i\beta} : \quad a \in \mathbb{C} \ni b{:} \quad \mathsf{L} \in \mathbb{C}^a$$

$$(a{:}b{:}\mathbb{L}) = \frac{a - ib \ \bigg|\ \mathbb{L}}{-\bar{\mathbb{L}}^* \ \bigg|\ a + ib}$$

$$\frac{a \ \bigg|\ \mathbb{L}}{-\bar{\mathbb{L}}^* \ \bigg|\ b} = \left(\frac{a+b}{2}{:}i\frac{a-b}{2}{:}\mathbb{L}\right)$$

$$\frac{a \ \bigg|\ \mathbb{L}}{-\bar{\mathbb{L}}^* \ \bigg|\ b} \ \frac{c \ \bigg|\ \mathbb{V}}{-\bar{\mathbb{V}}^* \ \bigg|\ d}^{\!*} \ \frac{a \ \bigg|\ \mathbb{L}}{-\bar{\mathbb{L}}^* \ \bigg|\ b} = \frac{a \ \bigg|\ \mathbb{L}}{-\bar{\mathbb{L}}^* \ \bigg|\ b} \ \frac{\bar{c} \ \bigg|\ -\bar{\mathbb{V}}}{\mathbb{V}^* \ \bigg|\ \bar{d}} \ \frac{a \ \bigg|\ \mathbb{L}}{-\bar{\mathbb{L}}^* \ \bigg|\ b} = \frac{a\bar{c} + \mathbb{L}\mathbb{V}^* \ \bigg|\ -a\bar{\mathbb{V}} + \mathbb{L}\bar{d}}{-\bar{\mathbb{L}}^*\bar{c} + b\mathbb{V}^* \ \bigg|\ \bar{\mathbb{L}}^*\bar{\mathbb{V}} + b\bar{d}} \ \frac{a \ \bigg|\ \mathbb{L}}{-\bar{\mathbb{L}}^* \ \bigg|\ b}$$

$$= \frac{\left(a\bar{c} + \mathbb{L}\mathbb{V}^*\right) a - \left(-a\bar{\mathbb{V}} + \mathbb{L}\bar{d}\right)\bar{\mathbb{L}}^* \ \bigg|\ \left(a\bar{c} + \mathbb{L}\mathbb{V}^*\right)\mathbb{L} + \left(-a\bar{\mathbb{V}} + \mathbb{L}\bar{d}\right) b}{\left(-\bar{\mathbb{L}}^*\bar{c} + b\mathbb{V}^*\right) a - \left(\bar{\mathbb{L}}^*\bar{\mathbb{V}} + b\bar{d}\right)\bar{\mathbb{L}}^* \ \bigg|\ \left(-\bar{\mathbb{L}}^*\bar{c} + b\mathbb{V}^*\right)\mathbb{L} + \left(\bar{\mathbb{L}}^*\bar{\mathbb{V}} + b\bar{d}\right) b}$$

$$= \frac{a\bar{c}a + \mathbb{L}\mathbb{V}^*a + a\bar{\mathbb{V}}\bar{\mathbb{L}}^* - \mathbb{L}\bar{d}\bar{\mathbb{L}}^* \ \bigg|\ a\bar{c}\mathbb{L} + \mathbb{L}\mathbb{V}^*\mathbb{L} - a\bar{\mathbb{V}}b + \mathbb{L}\bar{d}b}{-\bar{\mathbb{L}}^*\bar{c}a + b\mathbb{V}^*a - \bar{\mathbb{L}}^*\bar{\mathbb{V}}\bar{\mathbb{L}}^* - b\bar{d}\bar{\mathbb{L}}^* \ \bigg|\ -\bar{\mathbb{L}}^*\mathbb{L}\bar{c} + b\mathbb{V}^*\mathbb{L} + \bar{\mathbb{L}}^*\bar{\mathbb{V}}b + b\bar{d}b}$$

$$\left(\frac{a+b}{2}{:}\frac{b-a}{2i}{:}\mathbb{L}\right)\left(\frac{c+d}{2}{:}\frac{d-c}{2i}{:}\mathbb{V}\right)^{\!*}\left(\frac{a+b}{2}{:}\frac{b-a}{2i}{:}\mathbb{L}\right) = 2\underbrace{\left(\frac{a+b}{2}{:}\frac{b-a}{2i}{:}\mathbb{L}\right)\left(\frac{c+d}{2}{:}\frac{d-c}{2i}{:}\mathbb{V}\right)}_{}\left(\frac{a+b}{2}{:}\frac{b-a}{2i}{:}\mathbb{L}\right)$$

$$- \underbrace{\left(\frac{a+b}{2}{:}\frac{b-a}{2i}{:}\mathbb{L}\right)\left(\frac{a+b}{2}{:}\frac{b-a}{2i}{:}\mathbb{L}\right)^{\!*}}_{}\left(\frac{c+d}{2}{:}\frac{d-c}{2i}{:}\bar{\mathbb{V}}\right)$$

$$= 2\left(\frac{a+b}{2}\frac{c+d}{2} + \frac{b-a}{2i}\frac{d-c}{2i} + \mathbb{L}\bar{\mathbb{V}}^{*}\right)\left(\frac{a+b}{2}{:}\frac{b-a}{2i}{:}\mathbb{L}\right) - \left(\frac{a+b}{2}^2 + \frac{b-a}{2i}^2 + \mathbb{L}\bar{\mathbb{L}}^*\right)\left(\frac{c+d}{2}{:}\frac{d-c}{2i}{:}\bar{\mathbb{V}}\right)$$

$$= 2\left(\frac{a+b}{2}\frac{c+d}{2} + \frac{b-a}{2i}\frac{d-c}{2i} + \mathbb{L}\bar{\mathbb{V}}^{*}\right)\frac{\dfrac{a+b}{2} - i\dfrac{b-a}{2i} \ \bigg|\ \mathbb{L}}{-\bar{\mathbb{L}}^* \ \bigg|\ \dfrac{a+b}{2} + i\dfrac{b-a}{2i}}$$

$$- \left(\frac{a+b}{2}^2 + \frac{b-a}{2i}^2 + \mathbb{L}\bar{\mathbb{L}}^*\right)\frac{\dfrac{c+d}{2} - i\dfrac{d-c}{2i} \ \bigg|\ \bar{\mathbb{V}}}{-\mathbb{V}^* \ \bigg|\ \dfrac{c+d}{2} + i\dfrac{d-c}{2i}}$$

$$z = x + iy$$

$$u = \alpha x + \beta y$$

$$v = \gamma x + \delta y$$

$$\frac{\alpha \mid \beta}{\gamma \mid \delta} = \frac{1 \mid -1}{1 \mid 1} \, \frac{a \mid b}{c \mid d} = \frac{a+c \mid d-b}{b+d \mid a-c}$$

$$\underbrace{a+ib}\,\underbrace{x+iy} + \underbrace{c+id}\,\underbrace{x-iy} = \overbrace{\underbrace{a+c}_{=\alpha}x + \underbrace{d-b}_{=\beta}y} + i\,\overbrace{\underbrace{b+d}_{=\gamma}x + \underbrace{a-c}_{=\delta}y} = u + iv$$