$$K' = O\left(Z_{\mathbb{R}}\right)$$

$$\mathcal{S}_{\pm} = \mathbb{C}^{2^{m}}$$

$$a = 2m \text{ even : } \quad \varepsilon = 0$$

$$\mathcal{C}\left(Z_{\mathbb{R}}\right) = \mathbb{C}^{2^{2m+2}} = \mathbb{C}^{4^{m+1}} = \mathbb{C}^{2^{m+1}} \boxtimes \mathbb{C}^{2^{m+1}} \text{ simple}$$

$$\mathcal{S} = \mathbb{C}^{2^{m+1}} = \mathbb{C}^{2^{m}} \times \mathbb{C}^{2^{m}} = \mathcal{S}_{+} \times \mathcal{S}_{-} \text{ double}$$

$$\mathcal{S}_{\pm} = \mathbb{C}^{2^{m}}$$

$$a = 2m+1 \text{ odd : } \quad \varepsilon = 1$$

$$\mathcal{C}\left(Z_{\mathbb{R}}\right) = \mathbb{C}^{2^{2m+3}} = \mathbb{C}^{4^{m+1}} \times \mathbb{C}^{4^{m+1}} \text{ double}$$

$$\mathcal{C}_{\pm}\left(Z_{\mathbb{R}}\right) = \mathbb{C}^{4^{m+1}} = \mathbb{C}^{2^{m+1}} \boxtimes \mathbb{C}^{2^{m+1}} \text{ simple}$$

$$a = 1 \Longrightarrow \mathcal{C}\left(Z_{\mathbb{R}}\right) \underset{\text{double}}{=} \mathbb{C}^{8} = \mathbb{C}^{4} \times \mathbb{C}^{4}: \quad \mathcal{C}_{\pm}\left(Z_{\mathbb{R}}\right) = \mathbb{C}^{4} \underset{\text{simple}}{=} \mathbb{C}^{2} \boxtimes \mathbb{C}^{2} \Longrightarrow \mathcal{S} = \mathbb{C}^{2}$$

$$a = 2 \Longrightarrow \mathcal{C}\left(Z_{\mathbb{R}}\right) \underset{\text{simple}}{=} \mathbb{C}^{16} = \mathbb{C}^{4} \boxtimes \mathbb{C}^{4} \Longrightarrow \mathcal{S} = \mathbb{C}^{4} \underset{\text{double}}{=} \mathbb{C}^{2} \times \mathbb{C}^{2} = \mathcal{S}_{+} \times \mathcal{S}_{-}$$

$$a = 4 \Longrightarrow \mathcal{C}\left(Z_{\mathbb{R}}\right) \underset{\text{simple}}{=} \mathbb{C}^{64} = \mathbb{C}^{8} \boxtimes \mathbb{C}^{8} \Longrightarrow \mathcal{S} = \mathbb{C}^{8} \underset{\text{double}}{=} \mathbb{C}^{4} \times \mathbb{C}^{4} = \mathcal{S}_{+} \times \mathcal{S}_{-}$$