$$0 < a_n \searrow 0 \Longrightarrow \sum_{n}^{\mathbb{N}} (\overset{n}{-1})\, a_n \ \text{konv}$$

$$s_n = \sum_{n}^{0|n} (\overset{m}{-1})\, a_m$$

$$s_{2n} = \overbrace{a_0 - a_1} + \cdot\cdot + \overbrace{a_{2n-2} - a_{2n-1}} + a_{2n} \geqslant 0$$

$$s_{2n} \searrow$$

$$s_{2n+2} = s_{2n} - a_{2n+1} + a_{2n+2} = s_{2n} - \overbrace{a_{2n+1} - a_{2n+2}}^{\geqslant 0} \leqslant s_{2n}$$

$$s_{2n} \searrow s \geqslant 0$$

$$s_{2n+1} - s_{2n} = a_{2n+1} \searrow 0$$

$$s_{2n+1} \rightsquigarrow s$$

$$0 < a_n \searrow 0 \Longrightarrow \sum_n^{\mathbb{N}} (-1)^n a_n \ \text{konv}$$

$$0 \leqslant (-1)^m \underbrace{\sum_n^{\mathbb{N}} (-1)^n a_n - \sum_n^{m} (-1)^n a_n} \leqslant a_m$$

$$n \geqslant m \Longrightarrow 0 \leqslant (-1)^{m+1} \underbrace{s_n - s_m} \leqslant a_{m+1}$$

$$(-1)^{m+1} \underbrace{s_{m+2k} - s_m} = a_{m+1} - a_{m+2} + a_{m+3} - \cdot\cdot - a_{m+2k}$$

$$= \begin{cases} \overbrace{a_{m+1} - a_{m+2}} + \overbrace{a_{m+3} - a_{m+4}} + \cdot\cdot + \overbrace{a_{m+2k-1} - a_{m+2k}} \geqslant 0 \\ a_{m+1} - \underbrace{a_{m+2} - a_{m+3}} - \cdot\cdot - \underbrace{a_{m+2k-2} - a_{m+2k-1}} - a_{m+2k} \leqslant a_{m+1} \end{cases}$$

$$(-1)^{m+1} \underbrace{s_{m+2k+1} - s_m} = a_{m+1} - a_{m+2} + a_{m+3} - \cdot\cdot + a_{m+2k+1}$$

$$= \begin{cases} \overbrace{a_{m+1} - a_{m+2}} + \overbrace{a_{m+3} - a_{m+4}} + \cdot\cdot + \overbrace{a_{m+2k-1} - a_{m+2k}} + a_{m+2k+1} \geqslant 0 \\ a_{m+1} - \underbrace{a_{m+2} - a_{m+3}} - \cdot\cdot - \underbrace{a_{m+2k-2} - a_{m+2k-1}} - \underbrace{a_{m+2k} - a_{m+2k+1}} \leqslant a_{m+1} \end{cases}$$

$$n > m \Longrightarrow \overline{s_n - s_m} \leqslant a_{m+1} \Longrightarrow s_n \underset{\text{Cau}}{\rightsquigarrow}$$

$$\sum_{n \geqslant 1} \frac{(-1)^{n-1}}{n} \Rightarrow \log(2) = \int_1^2 \frac{dt}{t}$$

$$\sum_{n \geqslant 1} \frac{(-1)^{n-1}}{n^s} \Rightarrow \underbrace{1 - 2^{1-s}} \zeta(s)$$

$${}^{2}\not{\ell} = \underbrace{1 - 2^{1-1}}_{=0} \zeta \underbrace{(1)}_{=\infty}$$

$$\overline{\sum_n^m (-1)^n a_n} \leqslant a_m$$

$${}^{2M}\varsigma\, (-1)^{\cdot}\, a_{\cdot} = \sum_{n < 2M} (-1)^n a_n \leqslant \sum_{0 \leqslant n} (-1)^n a_n \leqslant \sum_{n \leqslant 2N} (-1)^n a_n = {}^{2N+1}\varsigma\, (-1)^{\cdot}\, a_{\cdot}$$

$$\sum_{n\leqslant 2N}\overset{n}{(-1)}\,a_n-\sum_{n}^{\mathbb{N}}\overset{n}{(-1)}\,a_n=-\sum_{2N<n}\overset{n}{(-1)}\,a_n\ \underset{\text{klammern}}{\overset{\text{rechts}}{=}}\ \underbrace{a_{2N+1}-a_{2N+2}}+\underbrace{a_{2N+3}-a_{2N+4}}+\cdots\geqslant 0$$

$$\sum_{n}^{\mathbb{N}}\overset{n}{(-1)}\,a_n-\sum_{n<2M}\overset{n}{(-1)}\,a_n=\sum_{2M\leqslant n}\overset{n}{(-1)}\,a_n\ \underset{\text{klammern}}{\overset{\text{links}}{=}}\ \underbrace{a_{2M}-a_{2M+1}}+\underbrace{a_{2M+2}-a_{2M+3}}+\cdots\geqslant 0$$

$$\begin{cases}0<a_n\searrow 0\\ M=\curlyvee_n\overline{{}^{n}\upharpoonleft\!\cdot\lightning}<\infty\end{cases}\quad\Longrightarrow\sum_n a_n\,{}^{n}\lightning\ \Rightarrow$$

$$\bigwedge_{\varepsilon}^{>0}\bigvee_{p}^{\mathbb{N}}2M\,\lightning^{p}\leqslant\varepsilon$$

$$\bigwedge_{0<p\leqslant q}\overline{\sum_n^{q+1\llcorner p}a_n\,{}^{n}\lightning}=\overline{\sum_n^{q+1\llcorner p}a_n\underbrace{{}^{n+1}\upharpoonleft\!\cdot\lightning-{}^{n}\upharpoonleft\!\cdot\lightning}}=\overline{\sum_n^{q+1\llcorner p}a_n\,{}^{n+1}\upharpoonleft\!\cdot\lightning-\sum_n^{q\llcorner p-1}a_{n+1}\,{}^{n+1}\upharpoonleft\!\cdot\lightning}$$

$$=\overline{\sum_n^{q\llcorner p}\underbrace{a_n-a_{n+1}}\,{}^{n+1}\upharpoonleft\!\cdot\lightning+a_q\,{}^{q+1}\upharpoonleft\!\cdot\lightning-a_p\,{}^{p}\upharpoonleft\!\cdot\lightning}\leqslant\sum_n^{q\llcorner p}\overline{\underbrace{a_n-a_{n+1}}\,{}^{n+1}\upharpoonleft\!\cdot\lightning}+\overline{a_q\,{}^{q+1}\upharpoonleft\!\cdot\lightning}+\overline{a_p\,{}^{p}\upharpoonleft\!\cdot\lightning}$$

$$=\sum_n^{q\llcorner p}\underbrace{a_n-a_{n+1}}_{\geqslant 0}\overset{\leqslant M}{\overline{{}^{n+1}\upharpoonleft\!\cdot\lightning}}+\underset{\geqslant 0}{a_q}\overset{\leqslant M}{\overline{{}^{q+1}\upharpoonleft\!\cdot\lightning}}+\underset{\geqslant 0}{a_p}\overset{\leqslant M}{\overline{{}^{p}\upharpoonleft\!\cdot\lightning}}\leqslant M\left(\sum_n^{q\llcorner p}\underbrace{a_n-a_{n+1}}+a_q+a_p\right)$$

$$=M\left(\underbrace{a_p-a_{p+1}}+\underbrace{a_{p+1}-a_{p+2}}+\cdots+\underbrace{a_{q-1}-a_q}+a_q+a_p\right)\underset{\text{telescop}}{=}2M\,a_p\leqslant\varepsilon\Longrightarrow\ \text{Cau}$$

$$0<a_n\searrow 0\ \underset{\text{series}}{\overset{\text{alt}}{\Longrightarrow}}\ \sum_n^{\mathbb{N}}\overset{n}{(-1)}\,a_n\succ$$

$${}^{n}\lightning=\overset{n}{(-1)}\Longrightarrow{}^{N}\upharpoonleft\,\overset{\cdot}{(-1)}=\begin{cases}0 & N\text{ even}\\ 1 & N\text{ odd}\end{cases}$$