K field abs value

$$K \ni a_n \rightsquigarrow a \Leftrightarrow \bigwedge_{\varepsilon}^{>0} \bigvee_{m}^{\mathbb{N}} \bigwedge_{n}^{\geqslant m} \ulcorner a_n - a \urcorner \leqslant \varepsilon \Leftrightarrow \bigwedge_{\varepsilon}^{>0} n \geqslant \frac{a}{\varepsilon} \curvearrowright \ulcorner a_n - a \urcorner \leqslant \varepsilon$$

$$a_n \rightsquigarrow a \Leftrightarrow a_n - a \rightsquigarrow 0$$

$$n \geqslant \frac{a}{\varepsilon} \Longrightarrow \ulcorner a_n \urcorner \leqslant \ulcorner a \urcorner + \varepsilon$$

$$\dot{a}_n \rightsquigarrow \dot{a} \overset{\text{sum}}{\underset{\text{rule}}{\Longrightarrow}} a_n + \acute{a}_n \rightsquigarrow a + \acute{a}$$

$$\frac{a + \acute{a}}{2\varepsilon} \leqslant \frac{a}{\varepsilon} \curlyvee \frac{\acute{a}}{\varepsilon}$$

$$n \geqslant \text{ RHS } \Longrightarrow \ulcorner \underbrace{a_n + \acute{a}_n} - \underbrace{a + \acute{a}} \urcorner = \ulcorner \underbrace{a_n - a} + \underbrace{\acute{a}_n - \acute{a}} \urcorner \underset{\text{trans}}{\leqslant} \ulcorner a_n - a \urcorner + \ulcorner \acute{a}_n - \acute{a} \urcorner \leqslant \varepsilon + \varepsilon = 2\varepsilon$$

$$\dot{a}_n \rightsquigarrow \dot{a} \overset{\text{prod}}{\underset{\text{rule}}{\Longrightarrow}} a_n \cdot \acute{a}_n \rightsquigarrow a \cdot \acute{a}$$

$$\frac{a\acute{a}}{\varepsilon} \leqslant \frac{a}{\varepsilon / \left(1 + \ulcorner a \urcorner + \ulcorner \acute{a} \urcorner\right)} \curlyvee \frac{\acute{a}}{1 \curlywedge \varepsilon / \left(1 + \ulcorner a \urcorner + \ulcorner \acute{a} \urcorner\right)}$$

$$n \geqslant \text{ RHS } \Longrightarrow \ulcorner a_n \acute{a}_n - a\acute{a} \urcorner = \ulcorner \underbrace{a_n - a} \acute{a}_n + a \underbrace{\acute{a}_n - \acute{a}} \urcorner \leqslant \ulcorner \underbrace{a_n - a} \acute{a}_n \urcorner + \ulcorner a \underbrace{\acute{a}_n - \acute{a}} \urcorner$$

$$\leqslant \ulcorner a_n - a \urcorner \ulcorner \acute{a}_n \urcorner + \ulcorner a \urcorner \ulcorner \acute{a}_n - \acute{a} \urcorner \leqslant \frac{\varepsilon}{1 + \ulcorner a \urcorner + \ulcorner \acute{a} \urcorner} \underbrace{\ulcorner \acute{a} \urcorner + 1} + \ulcorner a \urcorner \frac{\varepsilon}{1 + \ulcorner a \urcorner + \ulcorner \acute{a} \urcorner} = \varepsilon$$

$$a_n \leadsto a \neq 0 \overset{\text{quot}}{\underset{\text{rule}}{\Longrightarrow}} \bigwedge_{n}^{\text{fast}} a_n \neq 0 \wedge \frac{1}{a_n} \leadsto \frac{1}{a}$$

$$\frac{1}{n} \leqslant \left(\varepsilon \ulcorner a \urcorner^2/2\right)_a \curlywedge \left(\ulcorner a \urcorner/2\right)_a \Longrightarrow \ulcorner a_n \urcorner = \ulcorner a - \underbrace{a_n - a} \urcorner \geqslant \ulcorner a \urcorner - \ulcorner a_n - a \urcorner \geqslant \ulcorner a \urcorner - \frac{\ulcorner a \urcorner}{2} = \frac{\ulcorner a \urcorner}{2} \Longrightarrow \frac{1}{\ulcorner a_n \urcorner} \leqslant \frac{2}{\ulcorner a \urcorner}$$

$$\ulcorner \frac{1}{a_n} - \frac{1}{a} \urcorner = \ulcorner \frac{a_n - a}{a a_n} \urcorner = \ulcorner a_n - a \urcorner \frac{1}{\ulcorner a \urcorner} \frac{1}{\ulcorner a_n \urcorner} \leqslant \frac{\varepsilon \ulcorner a \urcorner^2}{2} \frac{1}{\ulcorner a \urcorner} \frac{2}{\ulcorner a \urcorner} = \varepsilon$$