

$$\sum_n^{\mathbb{N}} \overline{n} \llcorner < +\infty \Rightarrow \sum_n^{\mathbb{N}} \overline{n} \llcorner = \sum_n^{\mathbb{N}} \overline{\pi(n)} \llcorner$$

$$\bigwedge_{\varepsilon} \bigvee_{\text{fin } M} \bigwedge_{\text{fin } P} \overline{P} \llcorner \leq \varepsilon \Rightarrow \text{fin } M \cup \overline{\pi}^{-1} M \subset \mathbb{N}$$

$$M \cup \overline{\pi}^{-1} M \subset \cdot \subset \mathbb{N} \left\{ \begin{array}{l} N \cap M = M = \pi N \cap M \\ \text{fin } N \llcorner M \subset \mathbb{N} \llcorner M \\ \text{fin } \pi N \llcorner M \subset \mathbb{N} \llcorner M \end{array} \right.$$

$$\begin{aligned} \overline{N} \llcorner \llcorner - \overline{N} \llcorner \llcorner \pi &= \overline{N} \llcorner \llcorner - \overline{\pi N} \llcorner \llcorner = \underbrace{\overline{N \cap M} \llcorner \llcorner}_{{}^M \llcorner \llcorner} + \overline{N \llcorner M} \llcorner \llcorner - \underbrace{\overline{\pi N \cap M} \llcorner \llcorner}_{{}^M \llcorner \llcorner} - \overline{\pi N \llcorner M} \llcorner \llcorner \\ &= \overline{N \llcorner M} \llcorner \llcorner - \overline{\pi N \llcorner M} \llcorner \llcorner \leq \overline{N \llcorner M} \llcorner \llcorner + \overline{\pi N \llcorner M} \llcorner \llcorner \leq 2\varepsilon \end{aligned}$$