$\mathbb{K}$ abs field

$$\sum_{n} a_n \underset{\text{summ}}{\Rrightarrow} \Longrightarrow {}^{n}\!\lrcorner a = \sum_{m}^{n} a_m \rightsquigarrow\Leftrightarrow \bigwedge_{\varepsilon}^{>0} \bigvee_{m}^{\mathbb{N}} \overline{\sum_{n}^{\geqslant m} a_n} \leqslant \varepsilon$$

$$\sum_{n} a_n \Rrightarrow \underset{\text{Cau}}{\Longleftrightarrow} \bigwedge_{\varepsilon}^{>0} \bigvee_{n_0}^{\mathbb{N}} \bigwedge_{q>p}^{\geqslant n_0} \overline{{}^{q \llcorner p}\langle \cdot a} = \overline{\sum_{n}^{q \llcorner p} a_n} \leqslant \varepsilon$$

$$\sum_{n}^{q \llcorner p} a_n = {}^{q}\langle \cdot a - {}^{p}\langle \cdot a$$

$$\text{Add-Rule} \quad \sum_{n} \overbrace{a_n + b_n} = \sum_{n} a_n + \sum_{n} b_n$$

$$\text{Mult-Rule} \quad \sum_{n} \alpha\, a_n = \alpha \sum_{n} a_n$$

$$\text{Lin-Rule} \quad \sum_{n} \overbrace{\alpha a_n + \beta b_n} = \alpha \sum_{n} a_n + \beta \sum_{n} b_n$$

$$\sum_{n} a_n \Rrightarrow \underset{\text{test}}{\overset{\text{triv}}{\longrightarrow}} a_n \rightsquigarrow 0$$

$$a_n = {}^{n+1}\langle \cdot a - {}^{n}\langle \cdot a \rightsquigarrow s - s = 0$$

$$\sum_{n} \overline{a_n} < \infty \underset{\text{summ}}{\overset{\text{abs}}{\Longrightarrow}} \sum_{n} a_n \Rrightarrow$$

$$\sum_{n} \overline{a_n} < \infty \Longrightarrow \bigwedge_{\varepsilon}^{>0} \bigvee_{n_0}^{\mathbb{N}} \bigwedge_{q>p}^{\geqslant n_0} \sum_{n}^{q \llcorner p} \overline{a_n} \leqslant \varepsilon \Longrightarrow \overline{\sum_{n}^{q \llcorner p} a_n} \leqslant \sum_{n}^{q \llcorner p} \overline{a_n} \leqslant \varepsilon$$

$$\overline{\lim}\,\overline{\frac{a_{n+1}}{a_n}} < 1 \Rightarrow \sum_n^{\mathbb{N}} \overline{a_n} < \infty \text{ abs summ}$$

$$1 > \overline{\lim}\,\overline{\frac{a_{n+1}}{a_n}} \swarrow \curlyvee_n^{\geqslant m} \overline{\frac{a_{n+1}}{a_n}} \overset{\text{Pos}}{\underset{\text{Rule}}{\Longrightarrow}} \bigvee_m^{\mathbb{N}} b = \curlyvee_n^{\geqslant m} \overline{\frac{a_{n+1}}{a_n}} < 1 \Rightarrow \bigwedge_n^{\geqslant m} \overline{\frac{a_{n+1}}{a_n}} \leqslant b$$

$$\Rightarrow \bigwedge_n^{\geqslant m} \frac{\overline{a_n}}{\overline{a_m}} = \overline{\frac{a_n}{a_{n-1}} \cdots \frac{a_{m+1}}{a_m}} \leqslant b^{n-m} = b^n / b^m$$

$$\bigwedge_\varepsilon^{>0} \bigvee_\ell^{\geqslant 0} b^\ell \leqslant \varepsilon \frac{1-b}{\overline{a_m}}$$

$$\Rightarrow \bigwedge_{q>p}^{\geqslant m+\ell} \frac{b^m}{\overline{a_m}} \sum_n^{q \llcorner p} \overline{a_n} \leqslant \sum_n^{q \llcorner p} b^n = \sum_n^{q} b^n - \sum_n^{p} b^n = \frac{1-b^q}{1-b} - \frac{1-b^p}{1-b} = \frac{b^p - b^q}{1-b} \leqslant \frac{b^p}{1-b} \leqslant \frac{b^{m+\ell}}{1-b} \leqslant \frac{\varepsilon b^m}{\overline{a_m}}$$

$$\Rightarrow \sum_n^{q \llcorner p} \overline{a_n} \leqslant \varepsilon \Rightarrow \sum_n^{\mathbb{N}} \overline{a_n} \text{ Cau}$$

$$\underline{\lim}\,\overline{\frac{a_{n+1}}{a_n}} > 1 \Rightarrow \sum_n^{\mathbb{N}} a_n \text{ not summ}$$

$$1 < a = \underline{\lim}\,\overline{\frac{a_{n+1}}{a_n}} \backsim \curlywedge_n^{\geqslant m} \overline{\frac{a_{n+1}}{a_n}} \overset{\text{Pos}}{\underset{\text{Rule}}{\Longrightarrow}} \bigvee_k^{\mathbb{N}} \curlywedge_n^{\geqslant k} \overline{\frac{a_{n+1}}{a_n}} \geqslant 1$$

$$\Rightarrow \bigwedge_n^{\geqslant k} \overline{\frac{a_{n+1}}{a_n}} \geqslant 1 \Rightarrow \overline{a_n} \geqslant \overline{a_k} > 0 \Rightarrow a_n \not\to 0 \overset{\text{triv}}{\underset{\text{test}}{\Longrightarrow}} \sum_n^{\mathbb{N}} a_n \text{ not summ}$$