$$R \text{ arch ordered field}$$

$$\begin{cases} R \xleftarrow[\text{isoton}]{\lrcorner} \mathbb{N} & \Leftrightarrow a_m \leqslant a_n \curvearrowleft m \leqslant n \\ R \xleftarrow[\text{antiton}]{\lrcorner} \mathbb{N} & \Leftrightarrow a_m \geqslant a_n \curvearrowleft m \leqslant n \end{cases}$$

$$\begin{cases} n \cdot 1 & \text{isoton} \\ \frac{1}{n} & \text{antiton} \end{cases}$$

$$\text{EX } \frac{n+a}{n^2+bn+c} \searrow \Longleftarrow \begin{cases} 2a+1 & \geqslant 0 \\ ab+a-c & \geqslant 0 \end{cases}$$

$$R \text{ arch } \begin{cases} \text{no bes } a_n & \underset{\text{iso}}{\nearrow} \\ \text{nu bes } a_n & \underset{\text{anti}}{\searrow} \end{cases} \overset{\text{Monotonie}}{\underset{\text{Satz}}{\Longrightarrow}} a_n \underset{\text{Cau}}{\rightsquigarrow}$$

$$\text{OE } a_n \nearrow : \quad a_n \leqslant M$$

$$\lightning\ a_n \not\rightsquigarrow \Longrightarrow \bigvee_{\varepsilon}^{>0} \bigwedge_{n}^{\mathbb{N}} \bigvee_{n'' \geqslant n' \geqslant n} \overbrace{a_{n''} - a_{n'}}^{\geqslant 0} = \lceil a_{n''} - a_{n'} \rceil = a_{n''} \bullet a_{n'} \geqslant \varepsilon \Longrightarrow a_{n''} \geqslant a_{n'} + \varepsilon$$

$$\bigwedge_{k}^{\mathbb{N}} \bigvee_{j_k} a_{j_k} \geqslant a_0 + k\varepsilon$$

$$j_0 = 0$$

$$\bigvee_{n'' \geqslant n' \geqslant j_k} a_{n''} - a_{n'} \geqslant \varepsilon \Longrightarrow a_{n''} \geqslant a_{n'} + \varepsilon \underset{\text{Mon}}{\geqslant} a_{j_k} + \varepsilon \underset{\text{Ind}}{\geqslant} a_0 + k\varepsilon + \varepsilon = a_0 + (k+1)\varepsilon : \quad j_{k+} = n''$$

$$\text{arch } \bigvee_{k}^{\mathbb{N}} \frac{1}{k} < \frac{\varepsilon}{M - a_0} \leqslant +\infty \Longrightarrow k > \frac{M - a_0}{\varepsilon} \Longrightarrow a_{j_k} \geqslant a_0 + k\varepsilon > M \geqslant a_{j_k}\ \lightning$$

$$\text{EX } x_0 = 2, x_{n+1} = \frac{x_n^2+2}{2x_n} \underset{\text{ind}}{\Longrightarrow} \begin{cases} x_n \in \mathbb{Q} & \\ x_n^2 \leqslant 2 & \text{bes} \\ x_{n+1} \leqslant x_n & \text{antiton} \end{cases} \Longrightarrow \begin{cases} x_n \underset{\text{Cau}}{\rightsquigarrow} \\ x_n^2 \rightsquigarrow 2 \end{cases}$$

$$a_n \nearrow \text{ no bes } \Longrightarrow \dot{a}_{\mathbb{N}} = a_\infty$$

$$a_n \searrow \text{ nu bes } \Longrightarrow \underset{\bullet}{a}_{\mathbb{N}} = a_\infty$$

$$\lightning \bigvee_{m}^{\mathbb{N}} a_m > a_\infty \Longrightarrow \bigwedge_{n \geqslant m} a_n \geqslant a_m > a_\infty \Longrightarrow \overline{a_n - a_\infty} \geqslant a_m - a_\infty > 0 \Longrightarrow a_n \not\rightsquigarrow a_\infty \lightning$$

$$\bigwedge_{n}^{\mathbb{N}} a_n \leqslant a_\infty \Longrightarrow a_{\mathbb{N}} \leqslant a_\infty \Longrightarrow \dot{a}_{\mathbb{N}} \leqslant a_\infty$$

$$M < a_\infty \Longrightarrow \bigvee_{n_0} \bigwedge_{n \geqslant n_0} \overline{a_n - a_\infty} < a_\infty - M \Longrightarrow M < a_n \Longrightarrow M \notin a_{\mathbb{N}} [\Longrightarrow a_\infty = \dot{a}_{\mathbb{N}}$$