$$\mathbb{L} \ni {}_n\mathbb{L} \rightsquigarrow {}_\infty\mathbb{L} \Leftrightarrow \bigwedge_{\varepsilon}^{>0} \bigvee_{m}^{\mathbb{N}} \bigwedge_{n}^{\geqslant m} \left| {}_n\mathbb{L} - {}_\infty\mathbb{L} \right| \leqslant \varepsilon$$

$${}_n\dot{\mathbb{L}} \rightsquigarrow {}_\infty\dot{\mathbb{L}} \underset{\text{rule}}{\overset{\text{prod}}{\Longrightarrow}} {}_n\mathbb{L} \times {}_n\acute{\mathbb{L}} \rightsquigarrow {}_\infty\mathbb{L} \times {}_\infty\acute{\mathbb{L}}$$

$$\bigvee_{M}^{>0} \bigwedge_{n}^{\mathbb{N}} \left| {}_n\acute{\mathbb{L}} \right| \leqslant M$$

$$\bigwedge_{\varepsilon}^{>0} \bigvee_{m}^{\mathbb{N}} \bigwedge_{n}^{\geqslant m} \left| {}_n\dot{\mathbb{L}} - {}_\infty\dot{\mathbb{L}} \right| \leqslant \frac{\varepsilon}{M + \left| {}_\infty\mathbb{L} \right|}$$

$$\left| {}_n\mathbb{L} \times {}_n\acute{\mathbb{L}} - {}_\infty\mathbb{L} \times {}_\infty\acute{\mathbb{L}} \right| = \left| \underbrace{{}_n\mathbb{L} - {}_\infty\mathbb{L}} \times {}_n\acute{\mathbb{L}} + {}_\infty\mathbb{L} \times \underbrace{{}_n\acute{\mathbb{L}} - {}_\infty\acute{\mathbb{L}}} \right| \leqslant \left| \underbrace{{}_n\mathbb{L} - {}_\infty\mathbb{L}} \times {}_n\acute{\mathbb{L}} \right| + \left| {}_\infty\mathbb{L} \times \underbrace{{}_n\acute{\mathbb{L}} - {}_\infty\acute{\mathbb{L}}} \right|$$

$$\leqslant \left| {}_n\mathbb{L} - {}_\infty\mathbb{L} \right| \left| {}_n\acute{\mathbb{L}} \right| + \left| {}_\infty\mathbb{L} \right| \left| {}_n\acute{\mathbb{L}} - {}_\infty\acute{\mathbb{L}} \right| \leqslant \frac{\varepsilon}{M + \left| {}_\infty\mathbb{L} \right|} M + \left| {}_\infty\mathbb{L} \right| \frac{\varepsilon}{M + \left| {}_\infty\mathbb{L} \right|} = \varepsilon$$

$${}_n\mathbb{L} \rightsquigarrow {}_\infty\mathbb{L} \in \mathbb{L}^{\complement} \underset{\text{rule}}{\overset{\text{quot}}{\Longrightarrow}} \bigwedge_{n}^{\text{fast}} {}_n\mathbb{L} \in \mathbb{L}^{\complement} \wedge {}_n\mathbb{L}^{-1} \rightsquigarrow {}_\infty\mathbb{L}^{-1}$$

$$\bigvee_{m_0}^{\mathbb{N}} \bigwedge_{n}^{\geqslant m_0} \left| {}_n\mathbb{L} - {}_\infty\mathbb{L} \right| \leqslant \frac{1}{2 \left| {}_\infty\mathbb{L}^{-1} \right|}$$

$$\left| {}_n\mathbb{L} \right| = \left| {}_\infty\mathbb{L} - \underbrace{{}_n\mathbb{L} - {}_\infty\mathbb{L}} \right| \geqslant \left| {}_\infty\mathbb{L} \right| - \left| {}_n\mathbb{L} - {}_\infty\mathbb{L} \right| \geqslant \left| {}_\infty\mathbb{L} \right| - \frac{\left| {}_\infty\mathbb{L}^{-1} \right|^{-1}}{2} = \frac{\left| {}_\infty\mathbb{L} \right|}{2} \Longrightarrow \frac{1}{\left| {}_n\mathbb{L} \right|} \leqslant \frac{2}{\left| {}_\infty\mathbb{L} \right|}$$

$$\bigwedge_{\varepsilon}^{>0} \bigvee_{m}^{\geqslant m_0} \bigwedge_{n}^{\geqslant m} \left| {}_n\mathbb{L} - {}_\infty\mathbb{L} \right| \leqslant \frac{\varepsilon}{2 \left| {}_\infty\mathbb{L}^{-1} \right|^2}$$

$$\left| {}_n\mathbb{L}^{-1} - {}_\infty\mathbb{L}^{-1} \right| = \left| {}_n\mathbb{L}^{-1} \times \underbrace{{}_n\mathbb{L} - {}_\infty\mathbb{L}} \times {}_\infty\mathbb{L}^{-1} \right| = \left| {}_n\mathbb{L}^{-1} \right| \left| {}_n\mathbb{L} - {}_\infty\mathbb{L} \right| \left| {}_\infty\mathbb{L}^{-1} \right| \leqslant 2 \left| {}_\infty\mathbb{L}^{-1} \right| \frac{\varepsilon}{2 \left| {}_\infty\mathbb{L}^{-1} \right|^2} \left| {}_\infty\mathbb{L}^{-1} \right| = \varepsilon$$