$$\begin{cases} 0 < \varepsilon_n \searrow 0 \\ M = \sup_n \left| B_n \right| < \infty \end{cases} \Rightarrow \sum_{n \in \mathbb{N}} \varepsilon_n \, b_n \text{ converges}$$

$$\forall_{0 < p \leqslant q} \left| \sum_{n=p}^{q} \varepsilon_n b_n \right| = \left| \sum_{n=p}^{q} \varepsilon_n \underbrace{\left(B_{n+1} - B_n \right)} \right| = \left| \sum_{n=p}^{q} \varepsilon_n B_{n+1} - \sum_{n=p}^{q-1} \varepsilon_{n+1} B_{n+1} \right|$$

$$= \left| \sum_{n=p}^{q-1} \underbrace{\left(\varepsilon_n - \varepsilon_{n+1} \right)} B_{n+1} + \varepsilon_q B_{q+1} - \varepsilon_p B_p \right| \leqslant \sum_{n=p}^{q-1} \left| \underbrace{\left(\varepsilon_n - \varepsilon_{n+1} \right)} B_{n+1} \right| + \left| \varepsilon_q B_{q+1} \right| + \left| \varepsilon_p B_p \right|$$

$$= \sum_{n=p}^{q-1} \underbrace{\left(\varepsilon_n - \varepsilon_{n+1} \right)}_{\geqslant 0} \overbrace{\left| B_{n+1} \right|}^{\leqslant M} + \underbrace{\varepsilon_q}_{\geqslant 0} \overbrace{\left| B_{q+1} \right|}^{\leqslant M} + \underbrace{\varepsilon_p}_{\geqslant 0} \overbrace{\left| B_p \right|}^{\leqslant M} \leqslant M \overbrace{\left(\sum_{n=p}^{q-1} \underbrace{\left(\varepsilon_n - \varepsilon_{n+1} \right)} + \varepsilon_q + \varepsilon_p \right)}$$

$$= M \overbrace{\left(\underbrace{\varepsilon_p - \varepsilon_{p+1}} + \underbrace{\varepsilon_{p+1} - \varepsilon_{p+2}} + \cdots + \underbrace{\varepsilon_{q-1} - \varepsilon_q} + \varepsilon_q + \varepsilon_p \right)} \underset{\text{telescop}}{=} 2M \varepsilon_p \rightsquigarrow 0 \Rightarrow \text{Cau}$$