$$\llcorner \overset{c}{\triangleleft}_{\Lambda} = \frac{\llcorner \ni {}_{\lambda}\lrcorner \rightsquigarrow}{\bigwedge_{U}^{\mathcal{U}} \bigvee_{\lambda}^{\Lambda} \bigwedge_{\mu:\nu \geqslant \lambda} {}_{\mu}\lrcorner - {}_{\nu}\lrcorner \in U}$$

$$\llcorner \overset{\ell}{\triangleleft}_{\Lambda} = \frac{\llcorner \ni {}_{\lambda}\lrcorner \rightsquigarrow {}_{\infty}\lrcorner}{\bigwedge_{U}^{\mathcal{U}} \bigvee_{\lambda}^{\Lambda} \bigwedge_{\mu \geqslant \lambda} {}_{\mu}\lrcorner - {}_{\infty}\lrcorner \in U}$$

$$\llcorner \overset{\omega}{\triangleleft}_{\Lambda} = \frac{\llcorner \ni {}_{\lambda}\lrcorner \rightsquigarrow 0}{\bigwedge_{U}^{\mathcal{U}} \bigvee_{\lambda}^{\Lambda} \bigwedge_{\mu \geqslant \lambda} {}_{\mu}\lrcorner \in U}$$

$$\llcorner \ni {}_{n}\lrcorner \rightsquigarrow {}_{\infty}\lrcorner \Leftrightarrow \bigwedge_{\varepsilon}^{>0} \bigvee_{n_0}^{\mathbb{N}} \bigwedge_{n}^{\geqslant n_0} \ulcorner {}_{n}\lrcorner - {}_{\infty}\lrcorner \urcorner \leqslant \varepsilon$$

$${}_{n}\dot{\lrcorner} \rightsquigarrow {}_{\infty}\dot{\lrcorner} \overset{\text{sum}}{\underset{\text{rule}}{\Longrightarrow}} {}_{n}\lrcorner + {}_{n}\acute{\lrcorner} \rightsquigarrow {}_{\infty}\lrcorner + {}_{\infty}\acute{\lrcorner}$$

$$n \geqslant [\varepsilon\colon .\lrcorner] \curlyvee [\varepsilon\colon .\acute{\lrcorner}] \Longrightarrow \ulcorner \underbrace{{}_{n}\lrcorner + {}_{n}\acute{\lrcorner}} - \underbrace{{}_{\infty}\lrcorner + {}_{\infty}\acute{\lrcorner}} \urcorner = \ulcorner \underbrace{{}_{n}\lrcorner - {}_{\infty}\lrcorner} + \underbrace{{}_{n}\acute{\lrcorner} - {}_{\infty}\acute{\lrcorner}} \urcorner \underset{\text{trans}}{\leqslant} \overbrace{\ulcorner {}_{n}\lrcorner - {}_{\infty}\lrcorner \urcorner}^{\leqslant \varepsilon} + \overbrace{\ulcorner {}_{n}\acute{\lrcorner} - {}_{\infty}\acute{\lrcorner} \urcorner}^{\leqslant \varepsilon} \leqslant 2\varepsilon$$

$$\begin{cases} a_n \rightsquigarrow a \\ {}_{n}\lrcorner \rightsquigarrow {}_{\infty}\lrcorner \end{cases} \overset{\text{prod}}{\underset{\text{rule}}{\Longrightarrow}} a_n \; {}_{n}\lrcorner \rightsquigarrow a \; {}_{\infty}\lrcorner$$

$$\bigvee_{M}^{>0} \bigwedge_{n}^{\mathbb{N}} \ulcorner {}_{n}\lrcorner \urcorner \leqslant M$$

$$\bigwedge_{\varepsilon}^{>0} \bigvee_{n_0}^{\mathbb{N}} \bigwedge_{n}^{\geqslant n_0} \ulcorner a_n - a \urcorner \leqslant \frac{\varepsilon}{M + \ulcorner {}_{\infty}\lrcorner \urcorner} \geqslant \ulcorner {}_{n}\lrcorner - {}_{\infty}\lrcorner \urcorner$$

$$\ulcorner a_n {}_{n}\lrcorner - a {}_{\infty}\lrcorner \urcorner = \ulcorner \underbrace{a_n - a} {}_{n}\lrcorner + a \underbrace{{}_{n}\lrcorner - {}_{\infty}\lrcorner} \urcorner \leqslant \ulcorner \underbrace{a_n - a} {}_{n}\lrcorner \urcorner + \ulcorner a \underbrace{{}_{n}\lrcorner - {}_{\infty}\lrcorner} \urcorner$$

$$= \ulcorner a_n - a \urcorner \ulcorner {}_{n}\lrcorner \urcorner + \ulcorner a \urcorner \ulcorner {}_{n}\lrcorner - {}_{\infty}\lrcorner \urcorner \leqslant \frac{\varepsilon}{M + \ulcorner {}_{\infty}\lrcorner \urcorner} M + \ulcorner a \urcorner \frac{\varepsilon}{M + \ulcorner {}_{\infty}\lrcorner \urcorner} = \varepsilon$$

$$ {}_{\lambda}\dot{\mathtt{l}} \rightsquigarrow\longrightarrow a \quad {}_{\lambda}\mathtt{l} + \acute{a} \quad {}_{\lambda}\acute{\mathtt{l}} \rightsquigarrow $$

$$ \bigwedge_{U}^{\mathcal{U}} \bigvee_{V}^{\mathcal{U}} aV + \acute{a}V \subset U \Rightarrow \bigvee_{\dot{\lambda} \in \Lambda} \bigwedge_{\mu:\nu \geqslant \dot{\lambda}} {}_{\mu}\dot{\mathtt{l}} - {}_{\nu}\dot{\mathtt{l}} \in V \Rightarrow \bigvee_{\Lambda \ni \varkappa \geqslant \lambda:\acute{\lambda}} \Rightarrow \bigwedge_{\mu:\nu \geqslant \varkappa} $$

$$ \underbrace{a\,{}_{\mu}\mathtt{l} + \acute{a}\,{}_{\mu}\acute{\mathtt{l}}} - \underbrace{a\,{}_{\nu}\mathtt{l} + \acute{a}\,{}_{\nu}\acute{\mathtt{l}}} = a \underbrace{{}_{\mu}\mathtt{l} - {}_{\nu}\mathtt{l}} + \acute{a} \underbrace{{}_{\mu}\acute{\mathtt{l}} - {}_{\nu}\acute{\mathtt{l}}} \in aV + \acute{a}V \subset U $$

$$ {}_{\lambda}\mathtt{l} \rightsquigarrow {}_{\infty}\mathtt{l} \Rightarrow {}_{\lambda}\mathtt{l} \rightsquigarrow $$

$$ \bigwedge_{U}^{\mathcal{U}} \bigvee_{V}^{\mathcal{U}} V - V \subset U \Rightarrow \bigvee_{\lambda}^{\Lambda} \bigwedge_{\mu \geqslant \lambda} {}_{\mu}\mathtt{l} - {}_{\infty}\mathtt{l} \in V \Rightarrow \bigwedge_{\mu:\nu \geqslant \lambda} {}_{\mu}\mathtt{l} - {}_{\nu}\mathtt{l} = \underbrace{{}_{\mu}\mathtt{l} - {}_{\infty}\mathtt{l}} + \underbrace{{}_{\infty}\mathtt{l} - {}_{\nu}\mathtt{l}} \in V - V \subset U $$

$$ {}_{\lambda}\dot{\mathtt{l}} \rightsquigarrow {}_{\infty}\dot{\mathtt{l}} \Rightarrow a \quad {}_{\lambda}\mathtt{l} + \acute{a} \quad {}_{\lambda}\acute{\mathtt{l}} \rightsquigarrow a \quad {}_{\infty}\mathtt{l} + \acute{a} \quad {}_{\infty}\acute{\mathtt{l}} $$

$$ \bigwedge_{U}^{\mathcal{U}} \bigvee_{V}^{\mathcal{U}} aV + \acute{a}V \subset U \Rightarrow \bigvee_{\dot{\lambda} \in \Lambda} \bigwedge_{\nu \geqslant \dot{\lambda}} {}_{\nu}\dot{\mathtt{l}} - {}_{\infty}\dot{\mathtt{l}} \in V \Rightarrow \bigvee_{\Lambda \ni \mu \geqslant \lambda:\acute{\lambda}} $$

$$ \Rightarrow \bigwedge_{\nu \geqslant \mu} \underbrace{a\,{}_{\nu}\mathtt{l} + \acute{a}\,{}_{\nu}\acute{\mathtt{l}}} - \underbrace{a\,{}_{\infty}\mathtt{l} + \acute{a}\,{}_{\infty}\acute{\mathtt{l}}} = a \underbrace{{}_{\nu}\mathtt{l} - {}_{\infty}\mathtt{l}} + \acute{a} \underbrace{{}_{\nu}\acute{\mathtt{l}} - {}_{\infty}\acute{\mathtt{l}}} \in aV + \acute{a}V \subset U $$

$$ {}_{\lambda}\mathtt{l} \rightsquigarrow {}_{\infty}\mathtt{l} : a_{\lambda} \rightsquigarrow a \Rightarrow a_{\lambda} \quad {}_{\lambda}\mathtt{l} \rightsquigarrow a \quad {}_{\infty}\mathtt{l} $$

$$\mathtt{l}\ \text{treu}\ \Rightarrow {}_{\lambda}\mathtt{l} \rightsquigarrow {}_{\cdot}\mathtt{l} \Rightarrow {}_{\infty}\mathtt{l} = {}_{\infty}\acute{\mathtt{l}}$$

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$$\bigwedge_{U}^{\mathcal{U}} \bigvee_{V}^{\mathcal{U}} V - V \subset U \Rightarrow \bigvee_{\dot{\lambda} \in \Lambda} \bigwedge_{\mu \geqslant \dot{\lambda}} {}_{\mu}\mathtt{l} - {}_{\infty}\dot{\mathtt{l}} \in V \Rightarrow \bigvee_{\Lambda \ni \nu \geqslant \lambda : \acute{\lambda}}$$

$$\Rightarrow \bigwedge_{\mu \geqslant \nu} {}_{\infty}\mathtt{l} - {}_{\infty}\acute{\mathtt{l}} = \underbrace{{}_{\infty}\mathtt{l} - {}_{\mu}\mathtt{l}} + \underbrace{{}_{\mu}\mathtt{l} - {}_{\infty}\acute{\mathtt{l}}} \in -V + V \subset U \Rightarrow {}_{\infty}\mathtt{l} - {}_{\infty}\acute{\mathtt{l}} \in \bigcap \mathcal{U} = 0 \Rightarrow {}_{\infty}\mathtt{l} = {}_{\infty}\acute{\mathtt{l}}$$