$$ {}^{2}\mathbb{Z}_2^{\mathsf{C}} = \frac{\begin{array}{c|c} a & b \\ \hline c & d \end{array} \in {}^{2}\mathbb{Z}_2}{ad - bc = 1} $$

$$ {}^{2}\mathbb{Z}_2^{\mathsf{C}} \ni \begin{cases} S = \begin{array}{c|c} 0 & -1 \\ \hline 1 & 0 \end{array} \\ T = \begin{array}{c|c} 1 & 1 \\ \hline 0 & 1 \end{array} \end{cases} \Longrightarrow S^2 = \overset{\frown}{ST}^{3} = 1 $$

$$ {}^{2}\mathbb{Z}_2^{\mathsf{C}} = [S{:}T] $$

$$ {}^{2}\mathbb{Z}_2^{\mathsf{C}} \ni \begin{array}{c|c} a & b \\ \hline c & d \end{array} \underset{\text{OE}}{\Longrightarrow} 0 \leqslant c $$

$$ \text{Ind } c \geqslant 0 $$

$$ c = 0 \Longrightarrow ad = 1 \Longrightarrow a = d = \pm 1 \Longrightarrow \begin{array}{c|c} a & b \\ \hline 0 & d \end{array} = \begin{array}{c|c} 1 & \pm b \\ \hline 0 & 1 \end{array} = T^{\pm b} $$

$$ c \neq 0 \Longrightarrow \bigvee_{q}^{\mathbb{Z}} \bigvee_{r}^{c} d = cq + r $$

$$ \Longrightarrow \begin{array}{c|c} b - aq & -a \\ \hline r & -c \end{array} ST^q = \begin{array}{c|c} b - aq & -a \\ \hline r & -c \end{array} \begin{array}{c|c} 0 & -1 \\ \hline 1 & 0 \end{array} \begin{array}{c|c} 1 & q \\ \hline 0 & 1 \end{array} = \begin{array}{c|c} a & b - aq \\ \hline c & r \end{array} \begin{array}{c|c} 1 & q \\ \hline 0 & 1 \end{array} = \begin{array}{c|c} a & b \\ \hline c & cq + r \end{array} = \begin{array}{c|c} a & b \\ \hline c & d \end{array} $$

$$ 0 \leqslant r < c \underset{\text{Ind}}{\Longrightarrow} \begin{array}{c|c} b - aq & -a \\ \hline r & -c \end{array} \in < S{:}T > \Longrightarrow \begin{array}{c|c} a & b \\ \hline c & d \end{array} \in < S{:}T > $$