

$$\mathbb{N}^x \times \mathbb{N}^x \triangleleft \mathbb{K} = \begin{cases} \mathbb{N}^x \times \mathbb{N}^x \xrightarrow{\wr} \mathbb{K} \\ m \wr^n \neq 0 \curvearrowright m \in n \end{cases}$$

$$\wr * \wr \subseteq \mathbb{N}^x \times \mathbb{N}^x \triangleleft \mathbb{K} \xleftarrow[\text{bilin}]{*} \mathbb{N}^x \times \mathbb{N}^x \triangleleft \mathbb{K} \otimes \mathbb{N}^x \times \mathbb{N}^x \triangleleft \mathbb{K} \ni \wr : \wr$$

$$m \underbrace{\wr * \wr}^\ell = \sum_{m \prec n \prec \ell} m \wr^n \otimes n \wr^\ell$$