$$\text{cst } h^{\cdot} = \left[h^{0} \quad h^{n\text{-}1} \right] \in \mathbb{K}^n$$

$$\mathbb{I}_{n}\mathbb{K}^n = \frac{\mathbb{I} \xrightarrow[\text{n-stet diff}]{1} \mathbb{K}}{{}^{t}_{n}1 + h^{\cdot}\,{}^{t}_{\cdot}1 = 0 = {}^{t}_{n}1 + \sum_{j}^{n} h^{j}\,{}^{t}_{j}1} \text{ hom Lsg-Raum}$$

$$P(z) = {}^{z}h = z^n + \sum_{j}^{n} {}_{j}h\, z^j \in \mathbb{R}[z] \text{ poly}$$

$$P\left(\frac{d}{dt}\right) = \left(\frac{d}{dt}\right)^n + \sum_{j}^{n} {}_{j}h \left(\frac{d}{dt}\right)^j \in \mathbb{R}[\frac{d}{dt}] \text{ cst coeff diff oper}$$

$$\mathbb{I}_{n}\mathbb{K}^n = \begin{cases} \mathbb{I} \xrightarrow[\text{n-stet diff}]{1} \mathbb{K} \\ P\left(\frac{d}{dt}\right) {}^{t}1 = 0 \end{cases} = \text{ Ker } P\left(\frac{d}{dt}\right)$$

$$P\left(\frac{d}{dt}\right) Q\left(\frac{d}{dt}\right) = PQ\left(\frac{d}{dt}\right) = Q\left(\frac{d}{dt}\right) P\left(\frac{d}{dt}\right)$$

$$P(z) = \prod_{j}^{n} \left(z - \lambda_j\right) \text{ distinct simple zeros}$$

$$\Rightarrow \mathbb{R} \xrightarrow[\text{Fund-Syst}]{{}^{t}\mathbb{1}^{j} = e^{\lambda t_j}} \mathbb{C} \text{ complex solutions}$$

$$P\left(\frac{d}{dt}\right) = \prod_{k}^{n} \left(\frac{d}{dt} - \lambda_k\right)$$

$$\left(\frac{d}{dt} - \lambda_j\right) {}^{t}\mathbb{1}^{j} = {}^{t}\underline{\mathbb{1}}^{j} - \lambda_j\, {}^{t}\mathbb{1}^{j} = \lambda_j\, e^{\lambda t_j} - \lambda_j\, e^{\lambda t_j} = 0$$

$$P\left(\frac{d}{dt}\right) {}^{t}\mathbb{1}^{j} = \prod_{k}^{n} \left(\frac{d}{dt} - \lambda_k\right) {}^{t}\mathbb{1}^{j} = \prod_{k}^{n \llcorner j} \left(\frac{d}{dt} - \lambda_k\right) \overbrace{\left(\frac{d}{dt} - \lambda_j\right) {}^{t}\mathbb{1}^{j}}^{=0} = 0$$

$$\det {}^{0}_{i\_}\mathbb{1}^{j} = \det \begin{array}{c|c} {}^{0}_{0\_}\mathbb{1}^{0} & {}^{0}_{0\_}\mathbb{1}^{n-1} \\ \hline {}^{0}_{n-1\_}\mathbb{1}^{0} & {}^{0}_{n-1\_}\mathbb{1}^{n-1} \end{array} = \prod_{i>j} \left(\lambda_i - \lambda_j\right) \neq 0 \text{ Vandermonde}$$

$$\Longrightarrow {}^{t}\mathbb{1}^{j} = e^{\lambda t_j} \text{ frei auf } \mathbb{R}$$

$${}^{t}_{i\_}\mathbb{1}^{j} = \left(\frac{d}{dt}\right)^{i} e^{\lambda t_j} = \lambda_j^{i}\, e^{\lambda t_j} \Longrightarrow {}^{0}_{i\_}\mathbb{1}^{j} = \lambda_j^{i}$$

$$\Longrightarrow \det {}^{0}_{i\_}\mathbb{1}^{j} = \det \lambda_j^{i} = \det \begin{array}{c|c} {}_{0}\lambda^{0} & {}_{0}\lambda^{n-1} \\ \hline {}_{n-1}\lambda^{0} & {}_{n-1}\lambda^{n-1} \end{array} \underset{\text{Van}}{=} \prod_{i>j} \left(\lambda_i - \lambda_j\right)$$

$$P(z) = \prod_{j}^{n} \left(z - \lambda_j\right)^{m_j} \text{ multiplicity } m_j \geqslant 1$$

$$\Longrightarrow \sum_{j}^{n} m_j = n$$

$$\Longrightarrow \text{ Fund-Syst } \frac{{}^{t}\mathbb{1}^{j} = t^{m} e^{\lambda t_j}}{m \in m_j: \quad 0 \leqslant m < m_j} \mathbb{C} \text{ complex solutions}$$

$$P\left(\frac{d}{dt}\right) = \prod_{k}^{n} \left(\frac{d}{dt} - \lambda_k\right)^{m_k}$$

$$\left(\frac{d}{dt} - \lambda_j\right) {}^{t}u\, {}^{t}\mathbb{1}^{j} = {}^{t}\underline{u}\, {}^{t}\mathbb{1}^{j} + {}^{\lambda}u\, t_j\, {}^{t}\mathbb{1}^{j} - \lambda_j\, {}^{t}u\, {}^{t}\mathbb{1}^{j} = {}^{t}\underline{u}\, {}^{t}\mathbb{1}^{j}$$

$$\Longrightarrow \left(\frac{d}{dt} - \lambda_j\right)^{k} {}^{t}u\, {}^{t}\mathbb{1}^{j} = {}^{t}_{k\_}u\, {}^{t}\mathbb{1}^{j} = \left(\frac{d}{dt}\right)^{k} {}^{\curlywedge}u\, {}^{t}v^{j}$$

$$\Longrightarrow \left(\frac{d}{dt} - \lambda_j\right)^{m_j} t^{m}\, {}^{t}\mathbb{1}^{j} = \underbrace{\left(\frac{d}{dt}\right)^{m_j} t^{m}}_{=0 \Longleftarrow m < m_j} {}^{t}\mathbb{1}^{j} = 0$$

$$P\left(\frac{d}{dt}\right) t^{m}\, {}^{t}\mathbb{1}^{j} = \prod_{k}^{n} \left(\frac{d}{dt} - \lambda_k\right)^{m_k} t^{m}\, {}^{t}\mathbb{1}^{j} = \prod_{k}^{n \llcorner j} \left(\frac{d}{dt} - \lambda_k\right)^{m_k} \overbrace{P_j \left(\frac{d}{dt} - \lambda_j\right)^{m_j} t^{mt}\mathbb{1}^{j}}^{=0} = 0$$

$$\left(\frac{d}{dt}\right)^{i}_{t=0} t^m e^{\lambda t} = \begin{cases} i!\dfrac{\lambda^{i-m}}{(i-m)!} & m \leqslant i \\ 0 & m > i \end{cases}$$

$$e^{\lambda t} t^m = \sum_{0 \leqslant k} \frac{\lambda^k}{k!} t^{k+m} \Longrightarrow \left(\frac{d}{dt}\right)^{i} e^{\lambda t} t^m = \left(\frac{d}{dt}\right)^{i} \sum_{0 \leqslant k} \frac{\lambda^k}{k!} t^{k+m} = \sum_{0 \leqslant k \geqslant i-m} \frac{\lambda^k}{k!} (k+m) \cdot\cdot (k+m+1-i)\, t^{k+m-i}$$

$$\Longrightarrow \left(\frac{d}{dt}\right)^{i}_{t=0} e^{\lambda t} t^m = \begin{cases} i!\dfrac{\lambda^{i-m}}{(i-m)!} & m \leqslant i \\ 0 & m > i \end{cases}$$

$$\left(\frac{d}{dt}\right)^{i}_{t=0} e^{\lambda t} \frac{t^m}{m!} = \begin{cases} \dfrac{\lambda^{i-m}}{\begin{bmatrix} i \\ m \end{bmatrix}} & m \leqslant i \\ 0 & m > i \end{cases}$$