$$\underset{1}{\overset{\mathbb{S}}{\mathbb{Z}}} \blacktriangleleft_{0} \mathbb{C}^\times \xrightarrow[\asymp]{\mathrm{Ind}} \mathbb{Z}$$

$$\iota \in {}^{\mathbb{H}}\triangle_0 \mathbb{C}^\times \text{ closed } \Longrightarrow \bigvee \varphi \in {}^{\mathbb{H}}\triangle_0\mathbb{R}: \quad {}^t\iota = \overline{{}^t\iota}\, e^{it\varphi}$$

$$\bigvee_{\text{partition}} H = \bigcup_{0\leqslant k\leqslant \ell} {}_kH: \quad {}^{{}_kH}\iota \subset {}_k\overset{\text{rund}}{U} \subsetneq \mathbb{C}^\times: \quad \psi_k = \Re i\,\ell_k \in {}^{{}_kU}\triangle_\infty \mathbb{R}: \quad {}^t\varphi_k = {}^{{}^t\iota}\psi_k \text{ on } {}_kH$$

$$0\leqslant k<\ell \Longrightarrow {}^t\iota \in {}_kU \cap {}_{k+1}U \Longrightarrow {}^{{}_k{}^t\iota}\psi_k - {}^{{}_k{}^t\iota}\psi_{k+1} \in 2\pi\mathbb{Z} \underset{\text{OE}}{\Longrightarrow} {}^{{}_k{}^t\iota}\psi_k = {}^{{}_k{}^t\iota}\psi_{k+1} \Longrightarrow \begin{cases} \bigvee \varphi \in {}^{\mathbb{H}}\triangle_0\mathbb{R} \\ {}^{{}_kH}\varphi = \varphi_k \end{cases}$$

$$n = {}^0_\sharp\iota = \frac{{}^1\varphi - {}^0\varphi}{2\pi} \Longrightarrow \iota \underset{\text{htp}}{\exists} e^{2\pi i n} \in \mathbb{C}^\times$$

$$2\pi \Leftarrow {}^0_\sharp u = \int^{dz} \iota\frac{1}{z} = \sum_k \underbrace{{}^{{}_k{}^t\iota}\ell_k - {}^{{}_{k+1}{}^t\iota}\ell_k} = i\sum_k \underbrace{{}_k{}^t\varphi - {}_{k+1}{}^t\varphi} = i\,\underbrace{{}^1\varphi - {}^0\varphi}$$

$${}^t\Gamma_s = \underbrace{s + (1-s)\,\overline{{}^t\iota}}_{>0}\, \exp i\,(s2\pi nt + (1-s)\,{}^t\varphi)$$