

$$\mathbb{S} \begin{array}{c} \mathbb{Z} \\ \mathbb{Z} \end{array} \begin{array}{c} \blacktriangleleft \\ \blacktriangleright \end{array} \mathbb{C}^\times \xrightarrow[\cong]{\text{Ind}} \mathbb{Z}$$

$$\mathfrak{l} \in \mathbb{H} \begin{array}{c} \mathbb{H} \\ \mathbb{Z} \end{array} \begin{array}{c} \blacktriangleleft \\ \blacktriangleright \end{array} \mathbb{C}^\times \text{ closed} \Rightarrow \forall \varphi \in \mathbb{H} \begin{array}{c} \mathbb{H} \\ \mathbb{Z} \end{array} \mathbb{R}: \quad {}^t\mathfrak{l} = \overline{{}^t\mathfrak{l}} e^{i^t\varphi}$$

$$\bigvee_{\text{partition}} H = \bigcup_{0 \leq k \leq \ell} {}^k H: \quad {}^k H \mathfrak{l} \subset {}^k \overset{\text{rund}}{U} \subset \mathbb{C}^\times: \quad \psi_k = \Re i \ell_k \in {}^k U \begin{array}{c} U \\ \mathbb{Z} \end{array} \mathbb{R}: \quad {}^t\varphi_k = {}^t\mathfrak{l} \psi_k \text{ on } {}^k H$$

$$0 \leq k < \ell \Rightarrow {}^t\mathfrak{l} \in {}^k U \cap {}^{k+1} U \Rightarrow {}^{k^t\mathfrak{l}} \psi_k - {}^{k^t\mathfrak{l}} \psi_{k+1} \in 2\pi\mathbb{Z} \xrightarrow{\text{OE}} {}^{k^t\mathfrak{l}} \psi_k = {}^{k^t\mathfrak{l}} \psi_{k+1} \Rightarrow \begin{cases} \forall \varphi \in \mathbb{H} \begin{array}{c} \mathbb{H} \\ \mathbb{Z} \end{array} \mathbb{R} \\ {}^k H \hat{\varphi} = \varphi_k \end{cases}$$

$$n = {}^0\mathfrak{l} = \frac{{}^1\varphi - {}^0\varphi}{2\pi} \Rightarrow \mathfrak{l} \underset{\text{htp}}{\exists} e^{2\pi i n} \in \mathbb{C}^\times$$

$$2\pi \leftarrow {}^0\mathfrak{l} u = \int \mathfrak{l} \frac{dz}{z} = \sum_k \underbrace{{}^{k^t\mathfrak{l}} \ell_k - {}^{k+1^t\mathfrak{l}} \ell_k}_{= i \sum_k {}^{k^t\varphi} - {}^{k+1^t\varphi}} = i \underbrace{{}^1\varphi - {}^0\varphi}$$

$${}^t\Gamma_s = \underbrace{s + (1-s) \overline{{}^t\mathfrak{l}}}_{> 0} \exp i (s 2\pi n t + (1-s) {}^t\varphi)$$