$$\overparen{1+\frac{1}{n}}^{n} \nearrow e \swarrow \overparen{1+\frac{1}{n}}^{n+1}$$

$$a_n = \overparen{1+\frac{1}{n}}^{n} = \frac{\overparen{n+1}^{n}}{n} = \frac{\overparen{n+1}^{n}}{n^n} \text{ isoton}$$

$$\frac{a_n}{a_{n-1}} = \frac{\overparen{n+1}^{n}}{n^n}\frac{\overparen{n-1}^{n-1}}{n^{n-1}} = \frac{n+1}{n}\left(\frac{\underparen{n+1}\,\underparen{n-1}}{n^2}\right)^{n-1} = \frac{n+1}{n}\left(\frac{n^2-1}{n^2}\right)^{n-1}$$

$$= \frac{n+1}{n}\left(1-\frac{1}{n^2}\right)^{n-1} \underset{\text{Bern}}{>} \frac{n+1}{n}\underbrace{1-\frac{n-1}{n^2}} = \frac{n+1}{n}\frac{n^2-n+1}{n^2} = \frac{n^3+1}{n^3} > 1 \Longrightarrow a_n > a_{n-1}$$

$$b_n = \overparen{1+\frac{1}{n}}^{n+1} = \frac{\overparen{n+1}^{n+1}}{n} = \frac{\overparen{n+1}^{n+1}}{n^{n+1}} \text{ antiton}$$

$$\frac{b_{n-1}}{b_n} = \frac{n^n}{\overparen{n-1}^{n}}\frac{n^{n+1}}{\overparen{n+1}^{n+1}} = \frac{n}{n+1}\left(\frac{n^2}{\underparen{n-1}\,\underparen{n+1}}\right)^{n} = \frac{n}{n+1}\left(\frac{n^2}{n^2-1}\right)^{n}$$

$$= \frac{n}{n+1}\left(1+\frac{1}{n^2-1}\right)^{n} \underset{\text{Bin}}{>} \frac{n}{n+1}\underbrace{1+\frac{n}{n^2-1}} = \frac{n}{n+1}\frac{n^2+n-1}{n^2-1} = \frac{n^3+n^2-n}{n^3+n^2-n-1} > 1 \Longrightarrow b_{n-1} > b_n$$

$$b_n = a_n\frac{n+1}{n} > a_n \Longrightarrow a_n \nearrow a \leqslant b \swarrow b_n$$

$$b-a \leftsquigarrow b_n - a_n = \overparen{1+\frac{1}{n}}^{n+1} - \overparen{1+\frac{1}{n}}^{n} = \overparen{1+\frac{1}{n}}^{n}\underbrace{1+\frac{1}{n}-1} = \overparen{1+\frac{1}{n}}^{n}\frac{1}{n} \rightsquigarrow a\cdot 0 = 0 \Longrightarrow a = b$$

$$2 = a_1 \leqslant e \leqslant b_1 = 4$$

$$x \in \mathbb{R} \xrightarrow{\exp} \mathbb{R}\llcorner 0 \ni {}^{x}\mathfrak{e} = \sum_{0\leqslant n}\frac{x^n}{n!}$$

$${}^{x}\mathfrak{e} = \sum_{0\leqslant n}\frac{x^n}{n!} \Longrightarrow R = \infty$$

$$\mathbb{K} \xrightarrow[\text{diff}]{\exp} \mathbb{K}$$

$$\sum_{0 \leqslant n} \frac{\overline{x^n}}{n!} \leqslant \sum_{0 \leqslant n} \frac{\overline{x}^n}{n!} < \infty \Rightarrow \sum_{0 \leqslant n} \frac{x^n}{n!} \in \mathbb{K}$$

$$\frac{n!}{(n-m)!n^m} = \frac{n\,(n-1)\cdots(n-m+1)}{n^m} = \prod_j^m \left(1 - \frac{j}{n}\right) \underset{n}{\nearrow} 1$$

$$\sum_{k \leqslant m} \frac{\overline{x^m}}{m!} < \varepsilon \Rightarrow \sum_m^{0|n} \frac{x^m}{m!} - \left(1 + \frac{x}{n}\right)^n = \sum_m^{0|n} \left(\frac{x^m}{m!} - \binom{n}{m}\frac{x^m}{n^m}\right) = \sum_m^{0|n} \frac{x^m}{m!}\left(1 - \frac{n!}{(n-m)!n^m}\right)$$

$$= \sum_m^{0|n} \frac{x^m}{m!} \overbrace{1 - \prod_j^m \left(1 - \frac{j}{n}\right)} = \underbrace{\sum_m^{k|n} \frac{x^m}{m!} \overbrace{1 - \prod_j^m \left(1 - \frac{j}{n}\right)}^{\leqslant 1}}_{|\cdot| < \varepsilon} + \underbrace{\sum_m^{k} \frac{x^m}{m!} \overbrace{1 - \prod_j^m \left(1 - \frac{j}{n}\right)}^{\leadsto 0}}_{\leadsto 0} \leqslant \varepsilon$$

$${}^{x+y}\mathfrak{e} = {}^{x}\mathfrak{e}\ {}^{y}\mathfrak{e}$$

$${}^{x+y}\mathfrak{e} = \sum_{0 \leqslant n} \frac{\overline{x+y}^n}{n!} \underset{\text{binomi}}{=} \sum_{0 \leqslant n} \frac{1}{n!} \sum_m^{0|n} \binom{n}{m} x^m\, y^{n-m} = \sum_{0 \leqslant n} \frac{1}{n!} \sum_m^{0|n} \frac{n!}{m!\,(n-m)!} x^m\, y^{n-m}$$

$$= \sum_{0 \leqslant n} \sum_m^{0|n} \frac{x^m}{m!} \frac{y^{n-m}}{(n-m)!} \underset{\text{Fub}}{=} \sum_{0 \leqslant m} \sum_{m \leqslant n} \frac{x^m}{m!} \frac{y^{n-m}}{(n-m)!} \underset{n-m=k}{=} \sum_{0 \leqslant m} \sum_{0 \leqslant k} \frac{x^m}{m!} \frac{y^k}{k!} = \sum_{0 \leqslant m} \frac{x^m}{m!} \sum_{0 \leqslant k} \frac{y^k}{k!} = {}^{x}\mathfrak{e}\ {}^{y}\mathfrak{e}$$

$$\underset{-}{\exp} = \exp$$

$${}^{x}\underset{-}{\exp} = \sum_{1 \leqslant n} \frac{n x^{n-1}}{n!} = \sum_{1 \leqslant n} \frac{x^{n-1}}{(n-1)!} = {}^{x}\mathfrak{e}$$

$$ {}^{x+y}\mathfrak{e} = {}^{x}\mathfrak{e}\,{}^{y}\mathfrak{e} $$

$$ \partial_x\, {}^{x+y}\mathfrak{e}\,{}^{-x}\mathfrak{e} = {}^{x+y}\mathfrak{e}\,{}^{-x}\mathfrak{e} - {}^{x+y}\mathfrak{e}\,{}^{-x}\mathfrak{e} = 0 \Longrightarrow {}^{x+y}\mathfrak{e}\,{}^{-x}\mathfrak{e} = \text{ cst } = {}^{y}\mathfrak{e} $$

$$ {}^{x}\mathfrak{e}\,{}^{-x}\mathfrak{e} = {}^{0}\mathfrak{e} = 1 $$

$$ {}^{x}\mathfrak{e} \neq 0 $$

$$ {}^{o+x}\mathfrak{e} = \sum_{0 \leqslant n} \frac{x^{n o}\mathfrak{e}}{n!} = {}^{o}\mathfrak{e}\,{}^{x}\mathfrak{e} $$

$$ \partial^n \exp = \exp $$

$$ \overline{{}^{o+x}\mathfrak{e} - \sum_{n \in m} \frac{x^n}{n!}\partial_o^n \exp} = \overline{\frac{x^m}{m!}\partial_y^m \exp} \leqslant \curlyvee_{0|x} \frac{\overline{x^m}}{m!}\overline{\exp} \rightsquigarrow 0 $$

$$ \overline{{}^{x}\mathfrak{e} - 1} \leqslant {}^{\overline{x}}\mathfrak{e} - 1 \leqslant \overline{x}\,{}^{\overline{x}}\mathfrak{e} $$

$$ \overline{{}^{x}\mathfrak{e} - 1} = \overline{\sum_{n>0} \frac{x^n}{n!}} \leqslant \sum_{n>0} \frac{\overline{x^n}}{n!} = {}^{\overline{x}}\mathfrak{e} - 1 \leqslant \sum_{n>0} \frac{\overline{x}^n}{(n-1)!} = \overline{x}\,{}^{\overline{x}}\mathfrak{e} $$