

$$\mathbb{C} \cup \infty = \frac{\frac{z}{w}}{z \in \mathbb{C} \ni w: z \neq 0 \vee w \neq 0}$$

$$(S) = \frac{z \in \bar{\mathbb{C}}}{[1 \quad z^*] \frac{d \mid b^*}{b \mid a} \begin{bmatrix} 1 \\ z \end{bmatrix} = az^*z + bz^* + b^*z + d = 0}$$

$$\text{circle } r^2 = \overline{z+b}^2 = zz^* + b^*z + bz^* + b^*b \Rightarrow a = 1$$

$$d = b^*b - r^2 \Rightarrow b^*b - ad = r^2 > 0$$

$$\text{line } Ax + By + d = 0 \Rightarrow b = \frac{A + iB}{2} \wedge zb^* + z^*b + d = 0 \wedge a = 0 \Rightarrow b^*b - ad = \frac{A^2 + B^2}{4} > 0$$

$$\text{converse } az^*z + bz^* + b^*z + d = 0 \Rightarrow \begin{cases} A = b + b^* \wedge B = (b - b^*)/i & a = 0 \\ o = -b/a \wedge r^2 = o^2 - d/a = \frac{b^*b - ad}{a^2} > 0 & a \neq 0 \end{cases}$$

$$\bar{\mathbb{R}} = \frac{0 \mid i}{-i \mid 0}$$

$$i\bar{\mathbb{R}} = \frac{0 \mid -1}{-1 \mid 0}$$

$$\mathbb{T} = \frac{-1 \mid 0}{0 \mid 1}$$